Electric Machinery and Apparatus 1
AE1B14SP1

Miroslav Chomát
chomat@fel.cvut.cz
room B3-248

Transformers

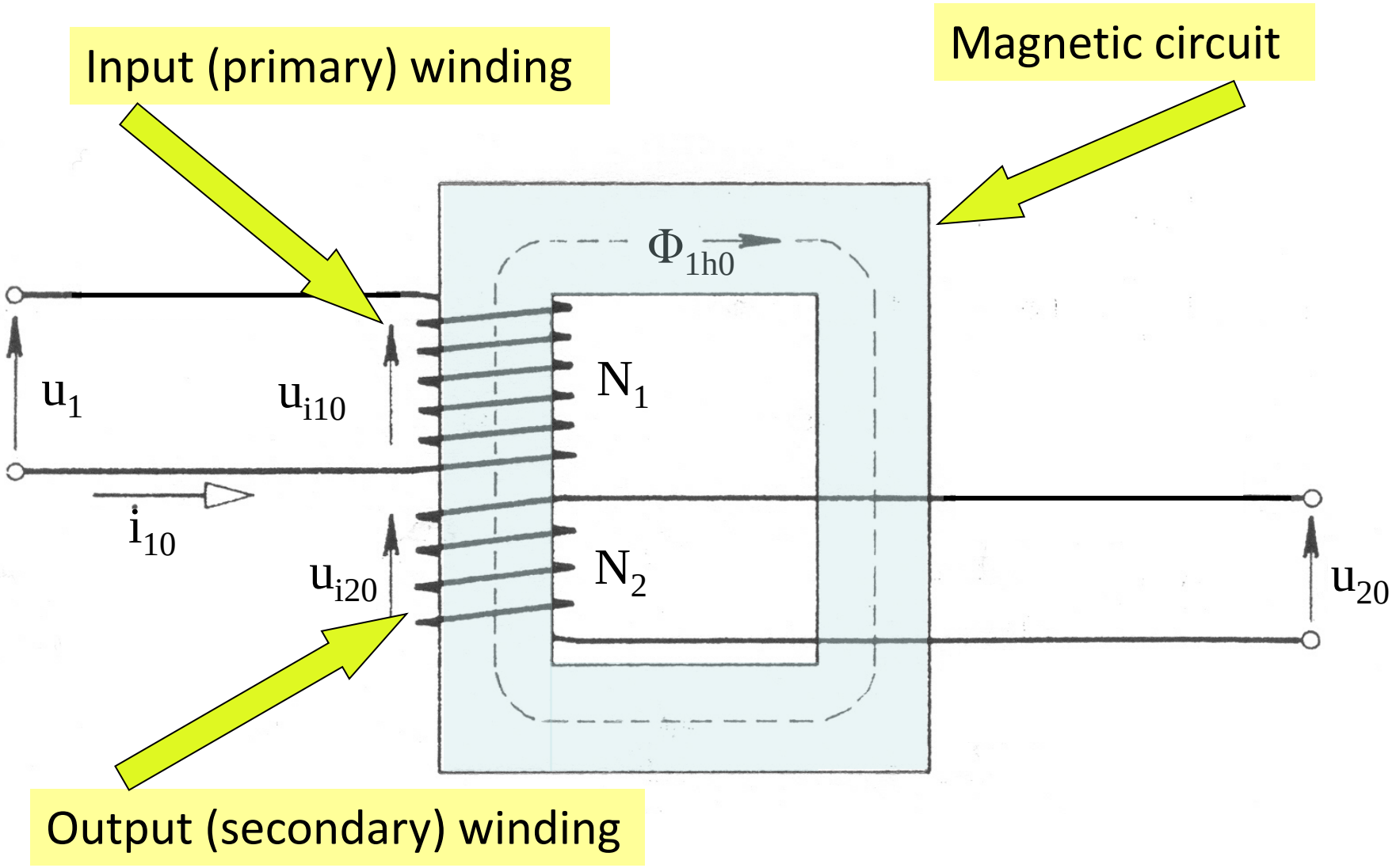
Change voltage, frequency remains

Principle of Function

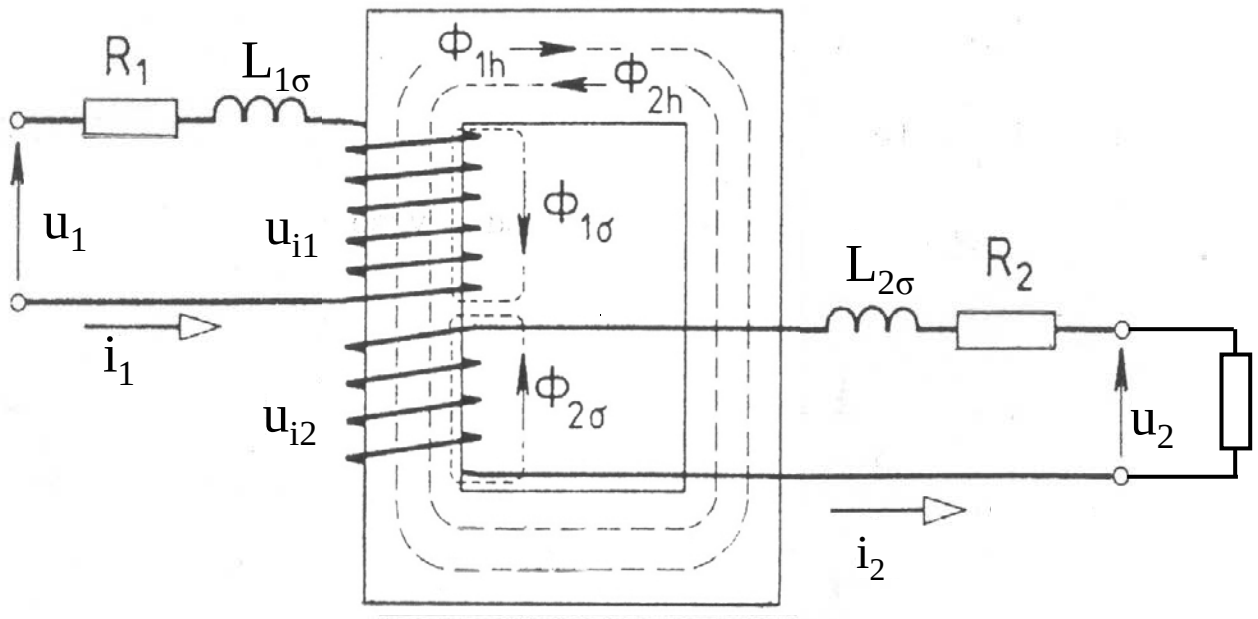
Maxwell-Faraday law of induction

$$-e = u_i = \frac{d\Psi}{dt} = N \cdot \frac{d\Phi}{dt}$$

Single-Phase Transformer



Fundamental Transformer Equation

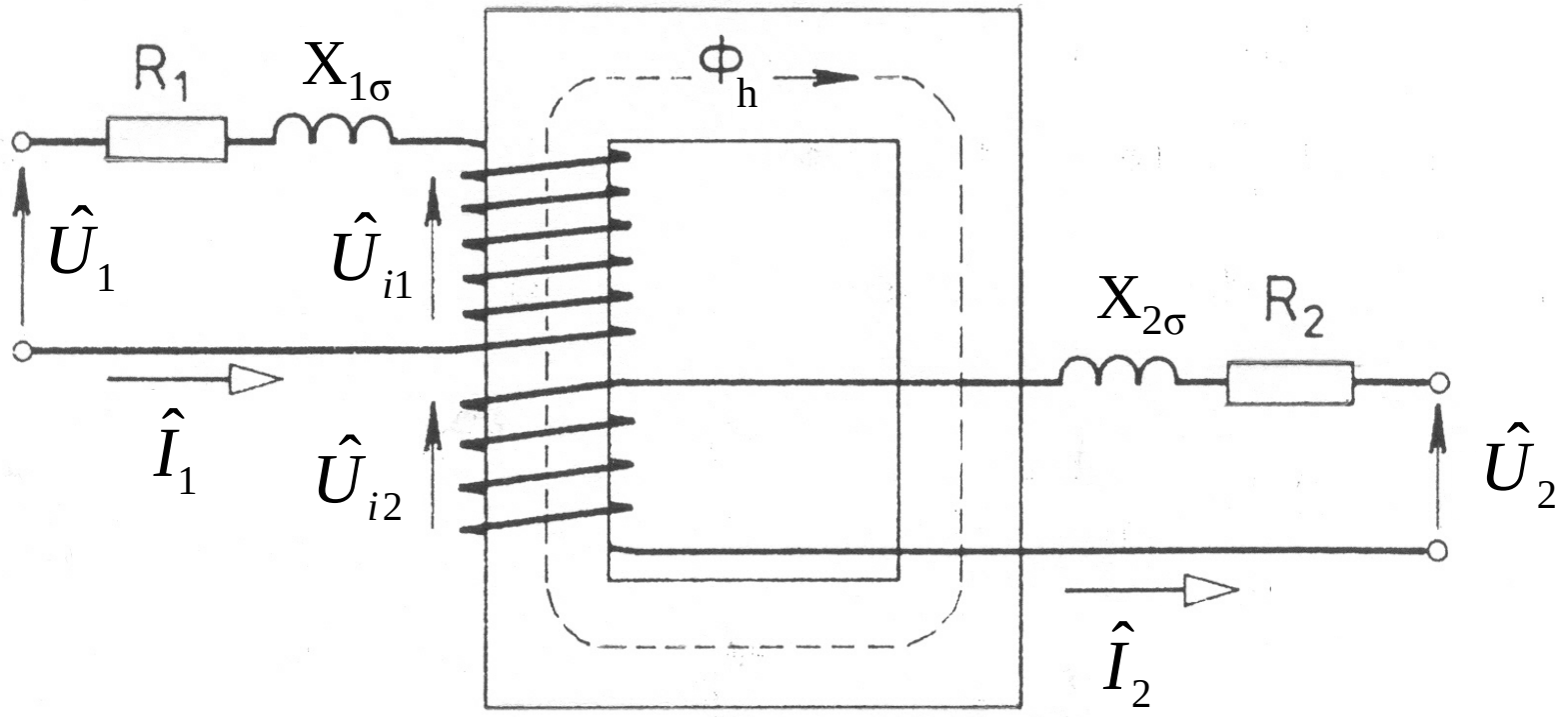


$$u_1 = R_1 \cdot i_1 + L_{1\sigma}\frac{di_1}{dt} + N_1\frac{d\Phi_\mu}{dt} = R_1 \cdot i_1 + L_{1\sigma}\frac{di_1}{dt} + u_{i1}$$

$$\Phi_\mu = \Phi_{1h} + \Phi_{2h}$$

$$u_2 = -R_2 \cdot i_2 - L_{2\sigma}\frac{di_2}{dt} + N_2\frac{d\Phi_\mu}{dt} = -R_2 \cdot i_2 - L_{2\sigma}\frac{di_2}{dt} + u_{i2}$$

Real Transformer with Sinusoidal Supply



$$\hat{U}_1 = R_1 \cdot \hat{I}_1 + j \cdot X_{1\sigma} \cdot \hat{I}_1 + \hat{U}_{i1}$$

$$\hat{U}_2 = -R_2 \cdot \hat{I}_2 - j \cdot X_{2\sigma} \cdot \hat{I}_2 + \hat{U}_{i2}$$

Electromotive Force U_i in Winding

$$\Phi_h = \Phi_{hm} \,.\, \sin \omega t$$

$$u_i = N \,.\, d\Phi / dt = N \,.\Phi_{hm} \,.\, \omega .\, \cos \omega t$$

$$U = \frac{U_m}{\sqrt{2}}$$ for sinusoidal waveform

$$U_{i1} = N_1 \cdot \Phi_{\mu \max} \cdot \omega \cdot \frac{1}{\sqrt{2}} \cdot 1$$

$$\omega = 2\pi f \qquad \frac{2\pi}{\sqrt{2}} = 4{,}44$$

$$U_{i1} = 4{,}44 \cdot f_1 \cdot N_1 \cdot \Phi_{\mu \max}$$

$$U_{i2} = 4{,}44 \cdot f_1 \cdot N_2 \cdot \Phi_{\mu \max}$$

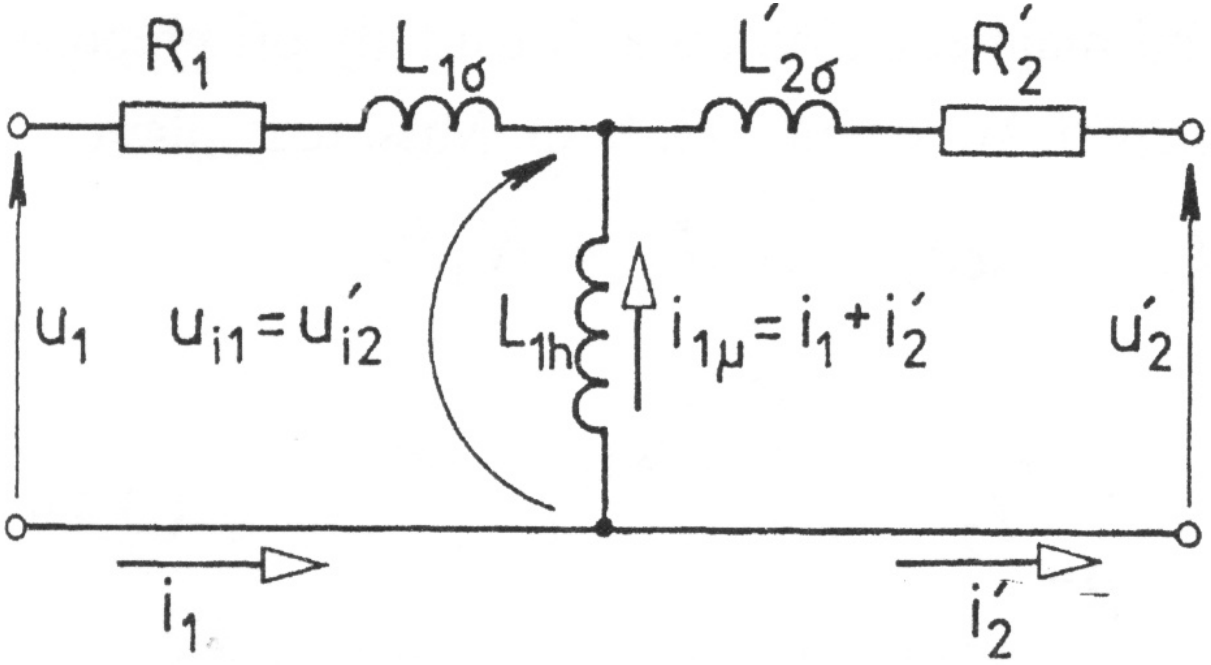
$$p = \frac{N_1}{N_2} = \frac{U_{i1}}{U_{i2}}$$

Watch out!

U_i … effective value

$\Phi_{\mu \max}$ … maximum value

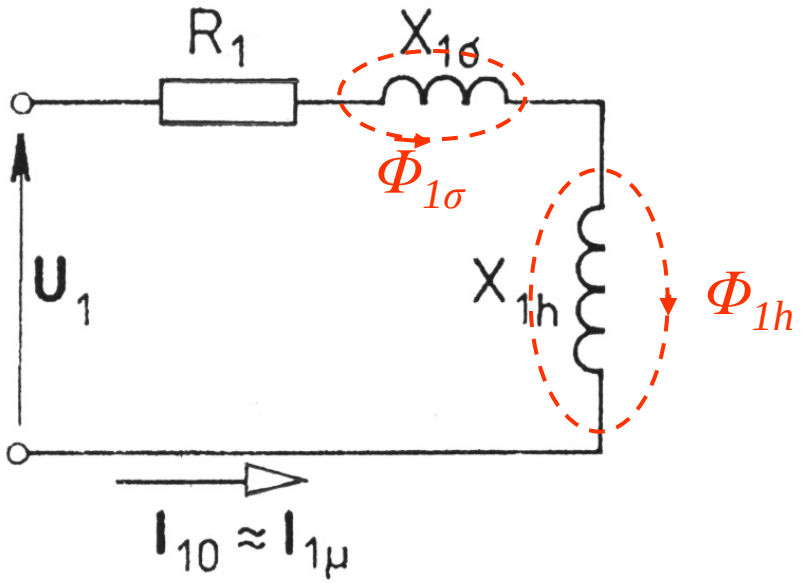
Equivalent Circuit of Transformer



No-Load Operation

$$\hat{U}_{10} = R_1 \cdot \hat{I}_{10} + jX_{1\sigma} + \hat{U}_{i10}$$

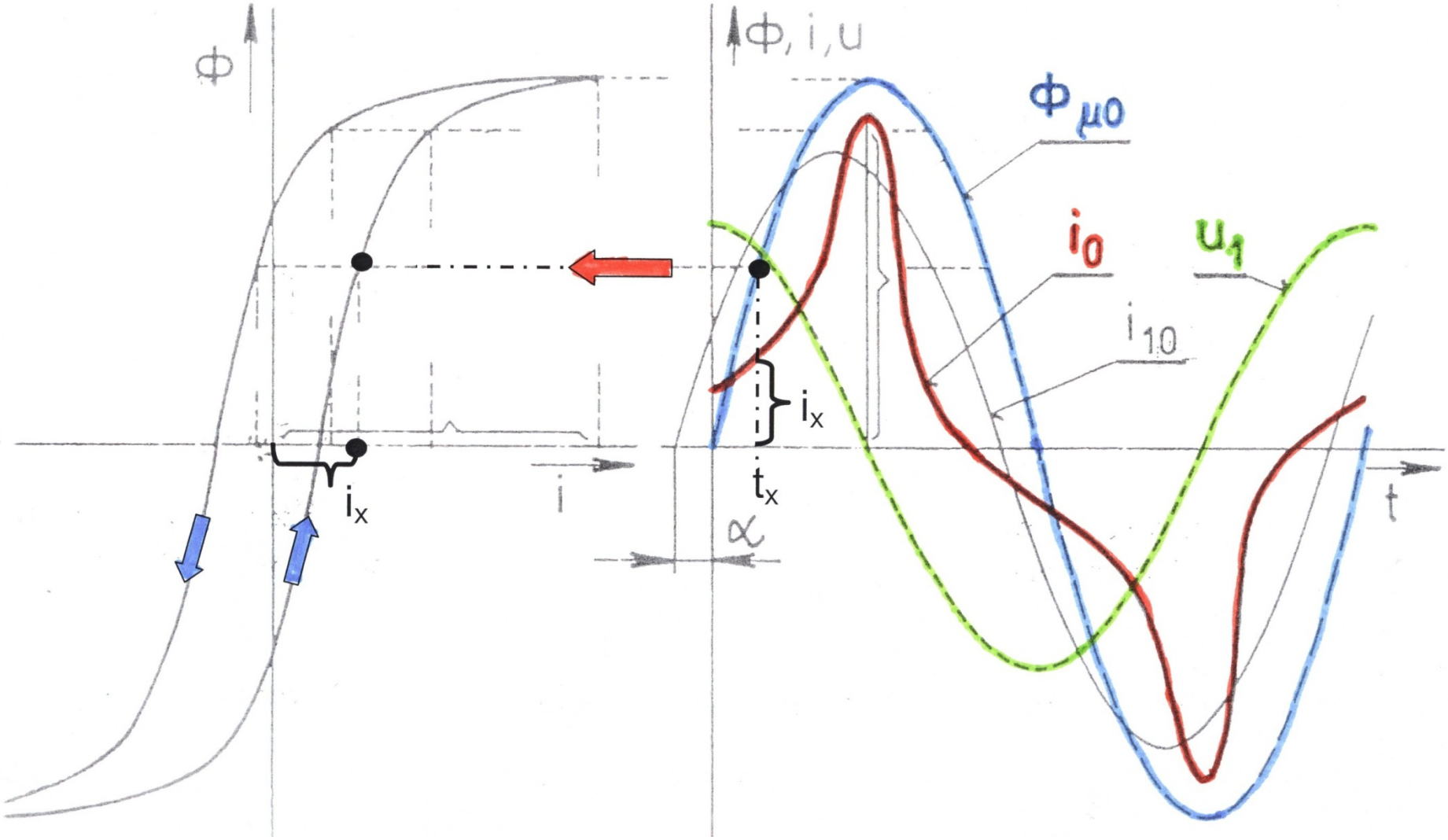
$$\hat{U}'_{20} = \hat{U}_{i10}$$



$$i_{10} \approx (5-10)\%$$

$$I_{10} = \frac{U_{10}}{\sqrt{R_1^2 + (X_{1\sigma} + X_{1h})^2}} \cong \frac{U_{10}}{X_{1h}} = I_\mu$$

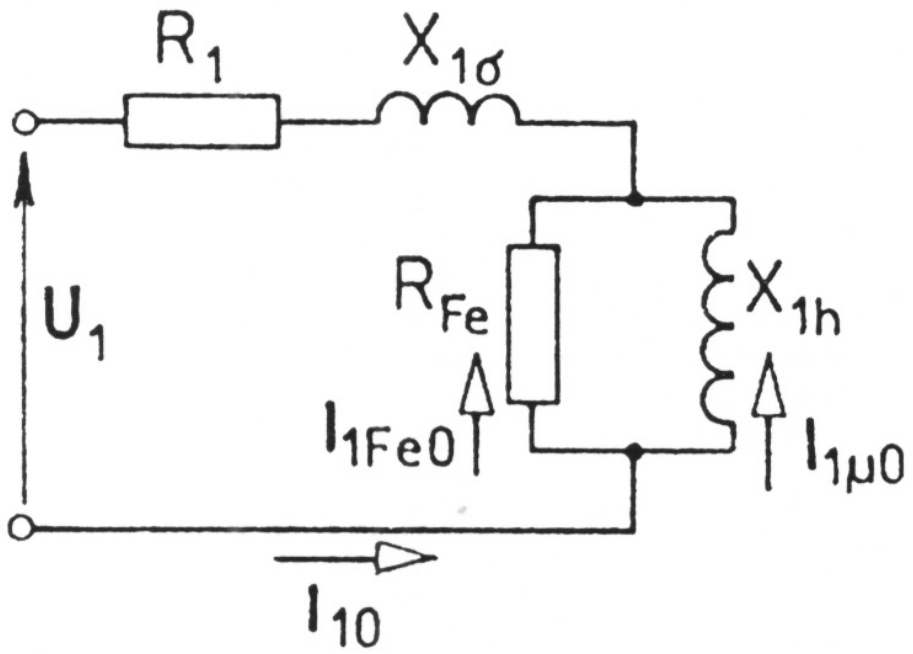
Current Waveform in No-Load Operation



Iron Losses ΔP_{Fe}

$\Delta P_{Fe} = f\,(B_{hm}, f)$ Losses in equivalent circuit are represented by appropriately placed resistor.

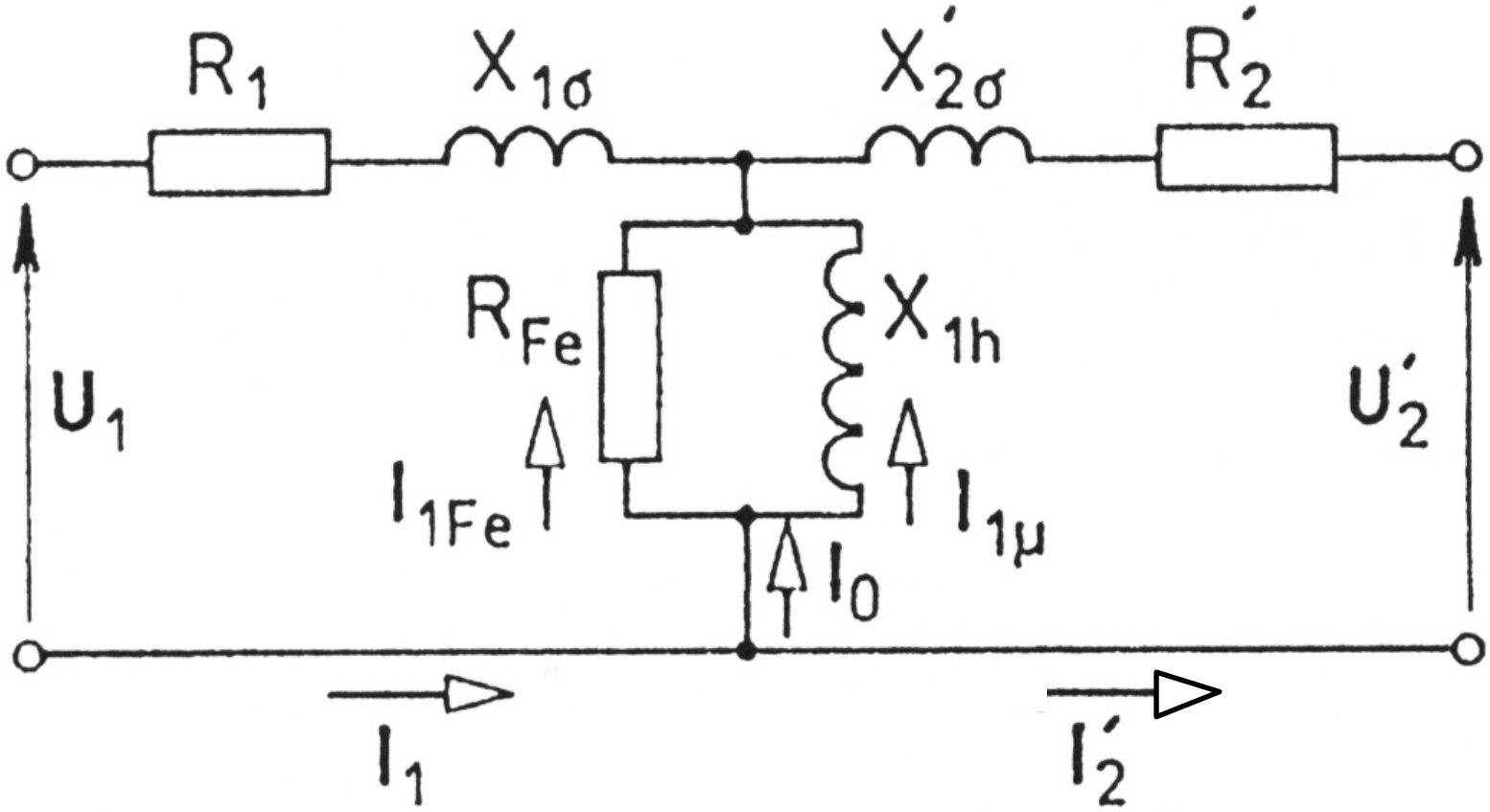
Equivalent resistance R_{Fe} is connected with main reactance X_{1h} in parallel.



$$P_{10} \approx \Delta P_{Fe0}$$

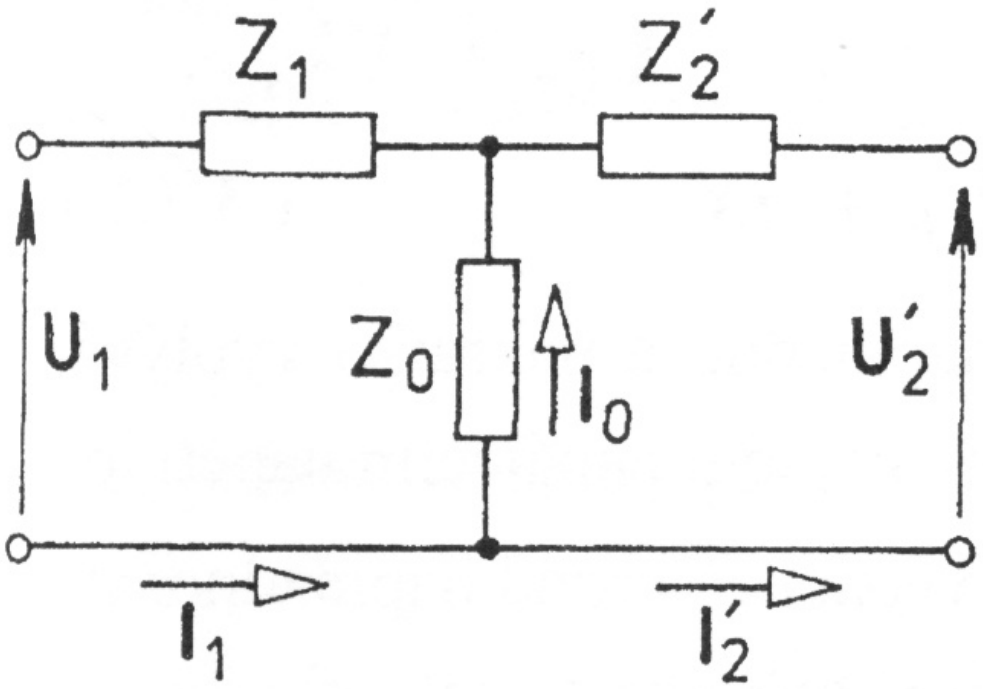
$$\hat{I}_{10} = \hat{I}_{1\mu} + \hat{I}_{1Fe}$$

Complete Equivalent Circuit of Transformer



$$\hat{I}_1 = \hat{I}_0 + \hat{I}'_2$$

Complete Equivalent Circuit of Transformer



$$\hat{Z}_0 = \frac{R_{Fe} \cdot jX_{1h}}{R_{Fe} + jX_{1h}}$$

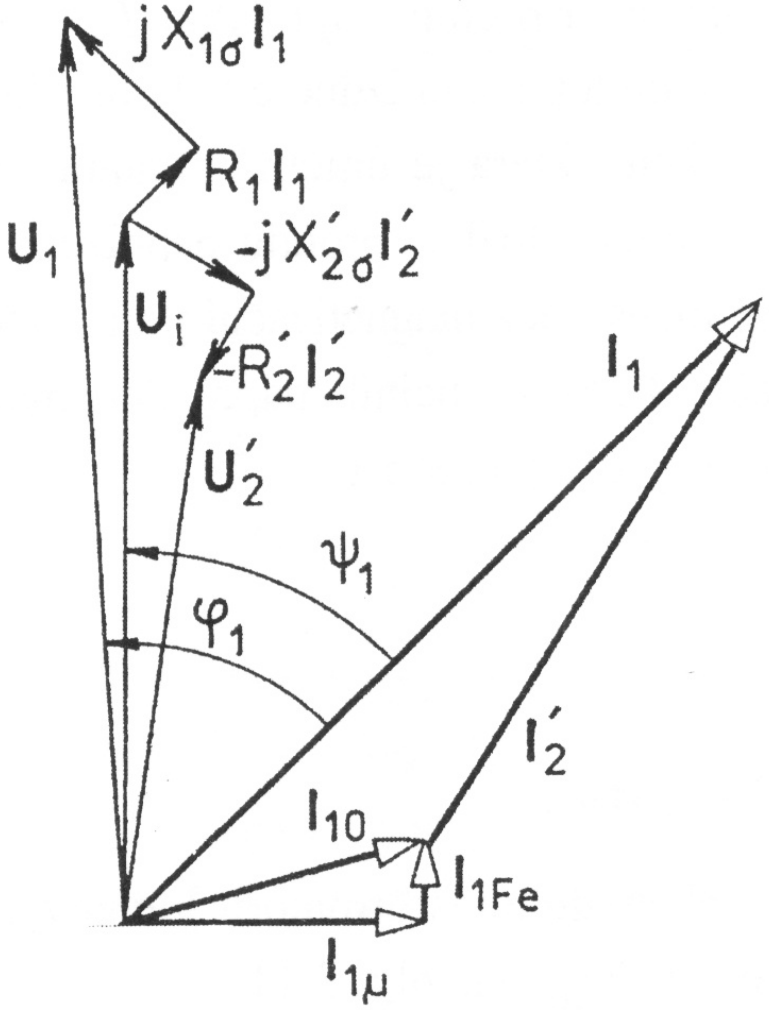
$$\hat{Z}_1 = R_1 + jX_{1\sigma}$$

$$\hat{Z}'_2 = R'_2 + jX'_{2\sigma}$$

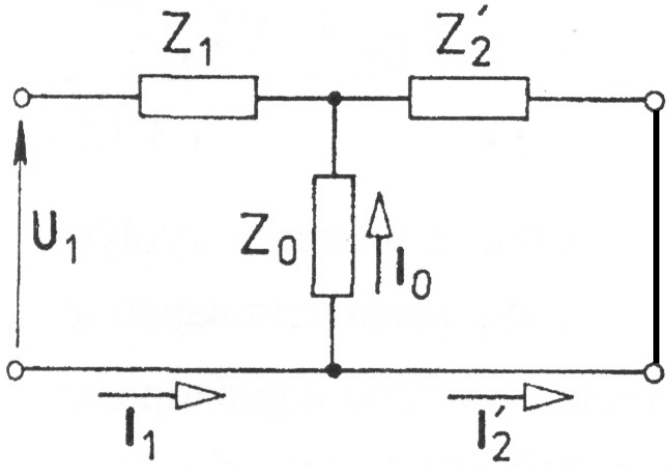
$$\hat{U}_1 = \hat{Z}_1 \cdot \hat{I}_1 + \hat{Z}_0 \cdot \hat{I}_0$$

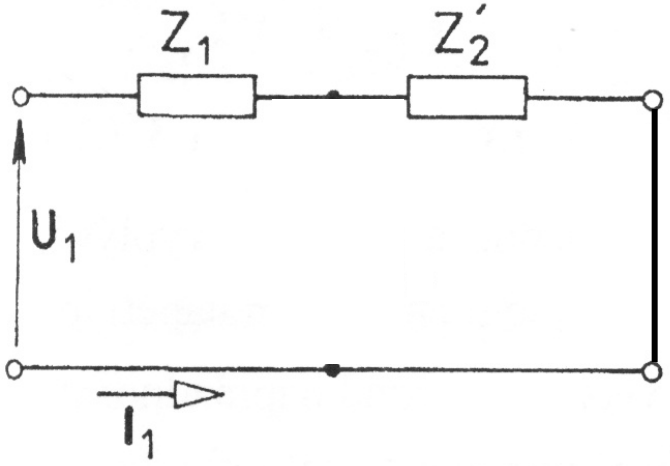
$$\hat{U}'_2 = -\hat{Z}'_2 \cdot \hat{I}'_2 + \hat{Z}_0 \cdot \hat{I}_0$$

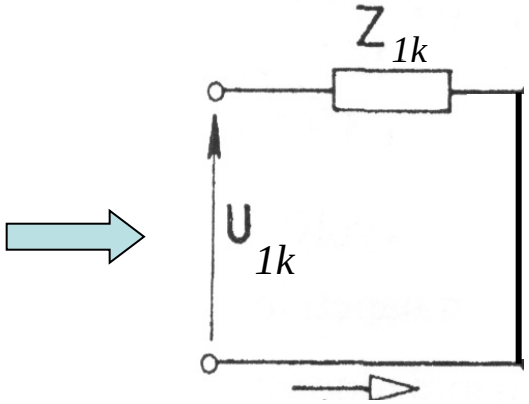
Phasor Diagram of Transformer with Load



Short-circuit operation







$I_{1k} = I_{1N}$

Short circuit voltage in per unit system:

$$u_{1k} = \frac{U_{1k}}{U_{1N}} = \frac{Z_{1k} \cdot I_{1N}}{U_{1N}} = \frac{Z_{1k}}{Z_{1N}} = z_{1k}$$

$$u_{1k} \approx (0{,}04 - 0{,}13)$$

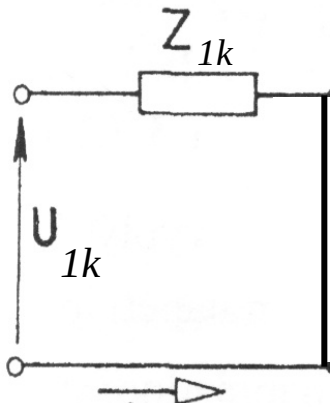
Short-circuit operation

Stable short-circuit current by nominal voltage:

$$I_{1k} = \frac{U_{1N}}{Z_{1k}} = \frac{U_{1N}/Z_{1N}}{Z_{1k}/Z_{1N}} = \frac{I_{1N}}{z_{1k}} = \frac{I_{1N}}{u_{1k}}$$

$$I_{1k} \approx (8-25) I_N$$

Short-Circuit Operation

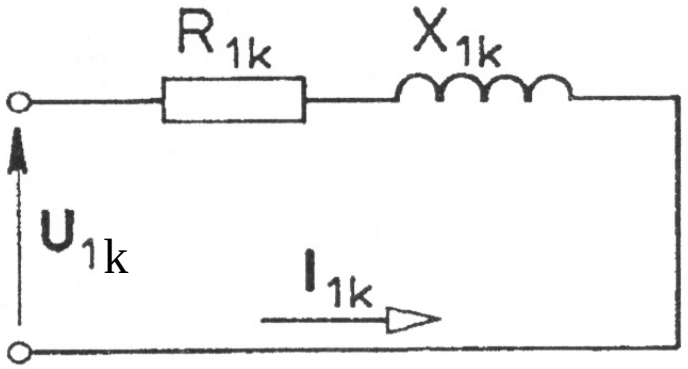


$I_{1k} = I_{1N}$

$$Z_{1k} = R_{1k} + jX_{1k}$$

$$R_{1k} = R_1 + R'_2$$

$$X_{1k} = X_{1\sigma} + X'_{2\sigma}$$



$$R_{1k} = Z_{1k} \cos \varphi_{1k}$$

$$X_{1k} = Z_{1k} \sin \varphi_{1k}$$

Per unit no-load losses Δp_{kN} :

$$r_{1k} = \frac{R_{1k}}{Z_{1N}} = \frac{R_{1k}}{Z_{1N}} \cdot \frac{I_{1N}^2}{I_{1N}^2} = \frac{\Delta P_{kN}}{S_N} = \Delta p_{kN}$$

Total short-circuit losses by nominal current:

$$\Delta P_{kN} = \Delta P_{j1N} + \Delta P_{j2N} + \Delta P_{dN}$$

Additional losses caused by leakage fluxes in construction and container.

(ΔP_{Fek} is neglected)

Losses and Efficiency

Losses within load operation:

$$\Delta P = \Delta P_{Fe} + \Delta P_{j1} + \Delta P_{j2} = \Delta P_0 + \Delta P_k$$

Efficiency:

$$\eta = \frac{P}{P_p} = \frac{P}{P + \Delta P_0 + \Delta P_k} = 1 - \frac{\Delta P_0 + \Delta P_k}{P + \Delta P_0 + \Delta P_k}$$

For per unit load ***i*** *:*

$$\eta = 1 - \frac{\Delta P_0 + i^2 \Delta P_{kN}}{i \cdot S_N \cos\varphi_2 + \Delta P_0 + i^2 \cdot \Delta P_{kN}}$$

Losses and Efficiency

Maximum efficiency by load operation is estimated from condition:

$$\frac{d\eta}{di} = 0 \quad \Rightarrow i^2 \Delta P_{kN} = \Delta P_k = \Delta P_0$$

Maximum η
occurs when losses in winding are equal to no-load losses.

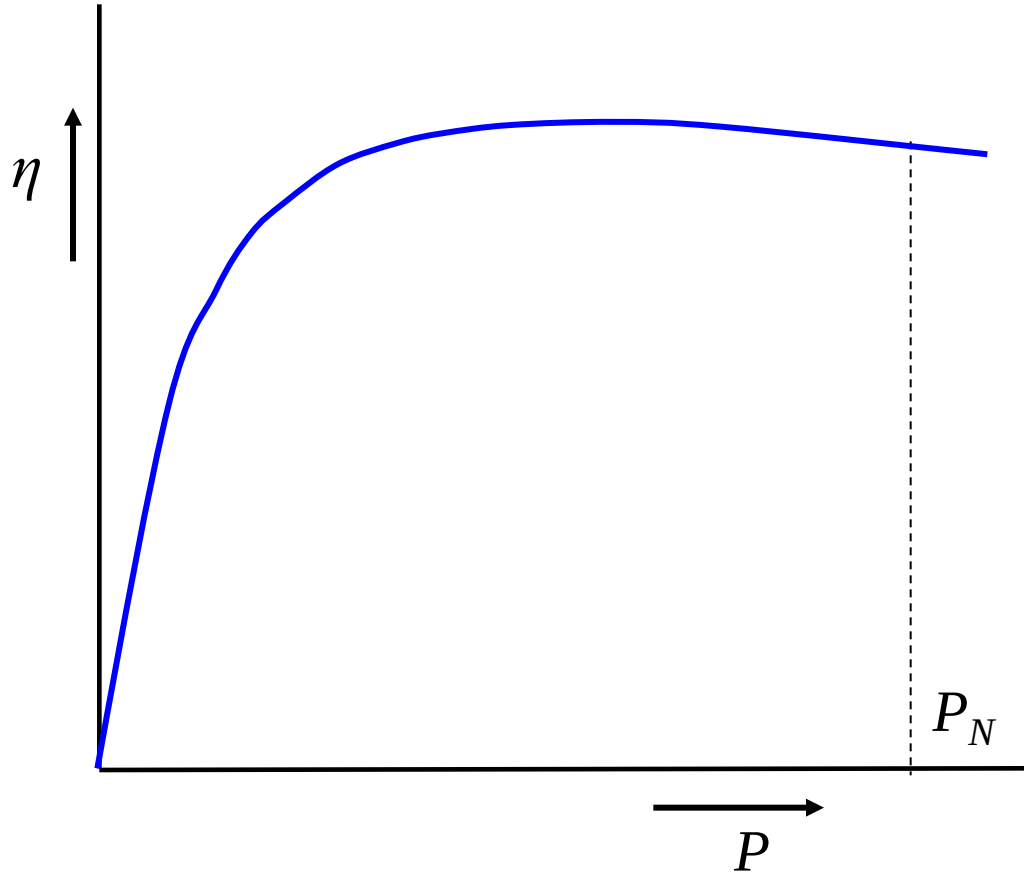
$$i = \sqrt{\frac{\Delta P_0}{\Delta P_{kN}}}$$

For a common value of losses $\Delta P_{kN} \approx 4 \cdot \Delta P_0$

we get maximum efficiency for per unit load $i \approx 0{,}5$

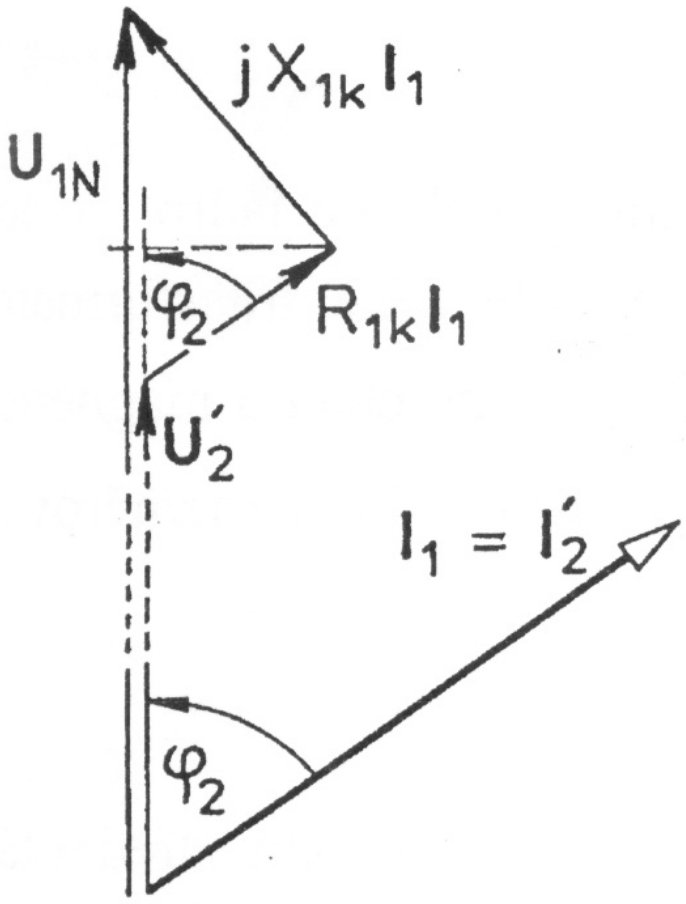
Losses and Efficiency

Typical waveform of transformer efficiency



Voltage Drop over Load

Is estimated as a difference between input voltage in no-load operation and input voltage by particular load current I_2 and $\cos\varphi_2$.

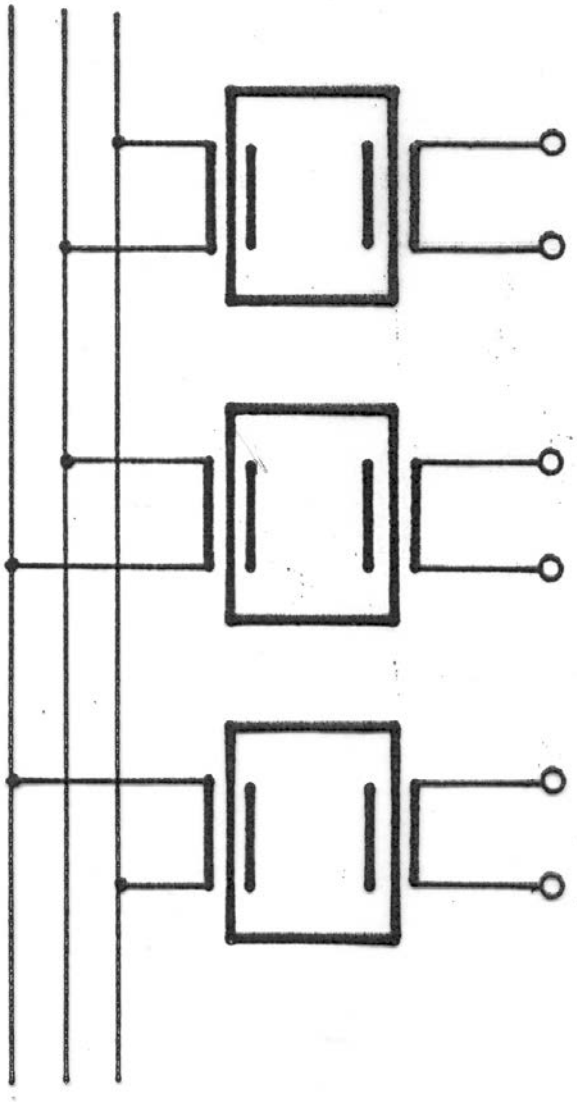


$$\Delta u \cong \frac{R_{1k} I_1 \cos\varphi_2 + X_{1k} I_1 \sin\varphi_2}{U_{1N}}$$

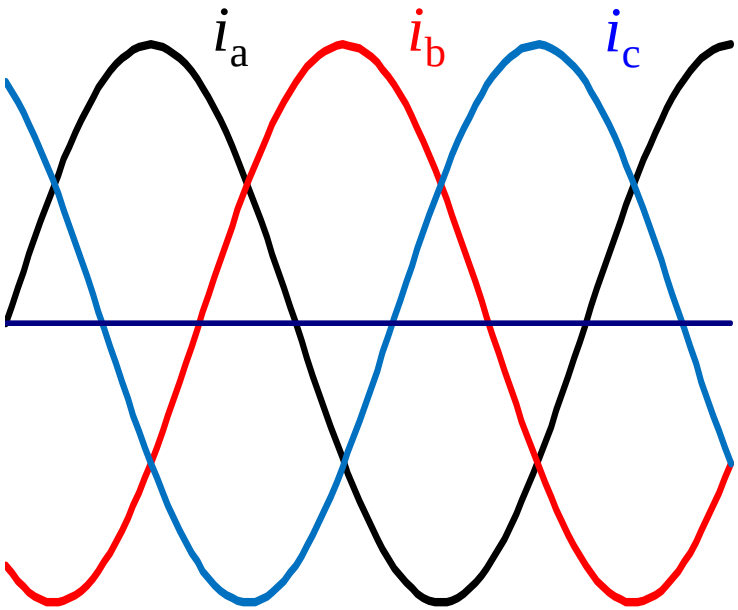
$$= \left(r_{1k} \cos\varphi_2 + x_{1k} \sin\varphi_2\right)\left(\frac{I_1}{I_{1N}}\right)$$

(Valid approximately for $z_{1k} < 4\ \%$)

Three-Phase Transformer with Independent Magnetic Circuit



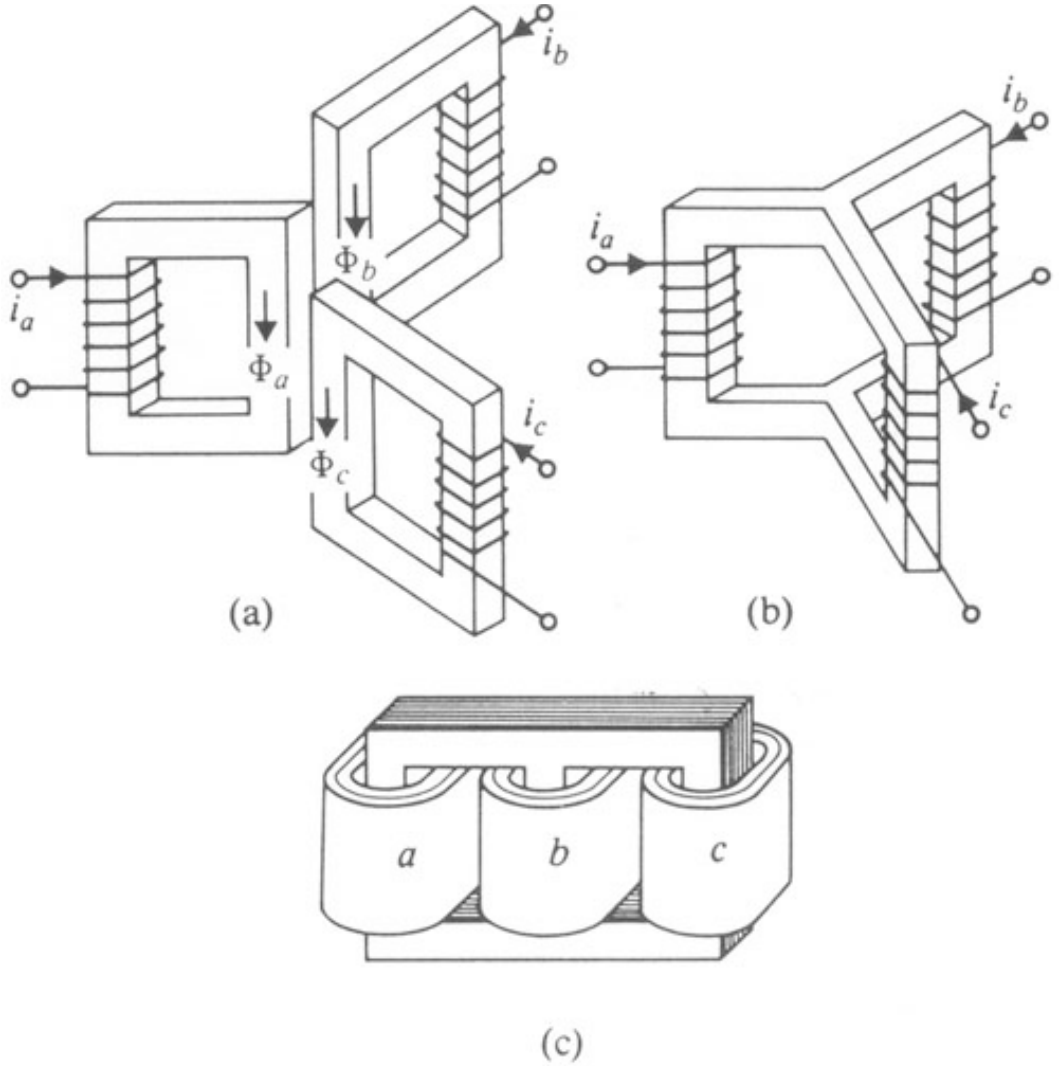
Symmetrical Three-Phase System



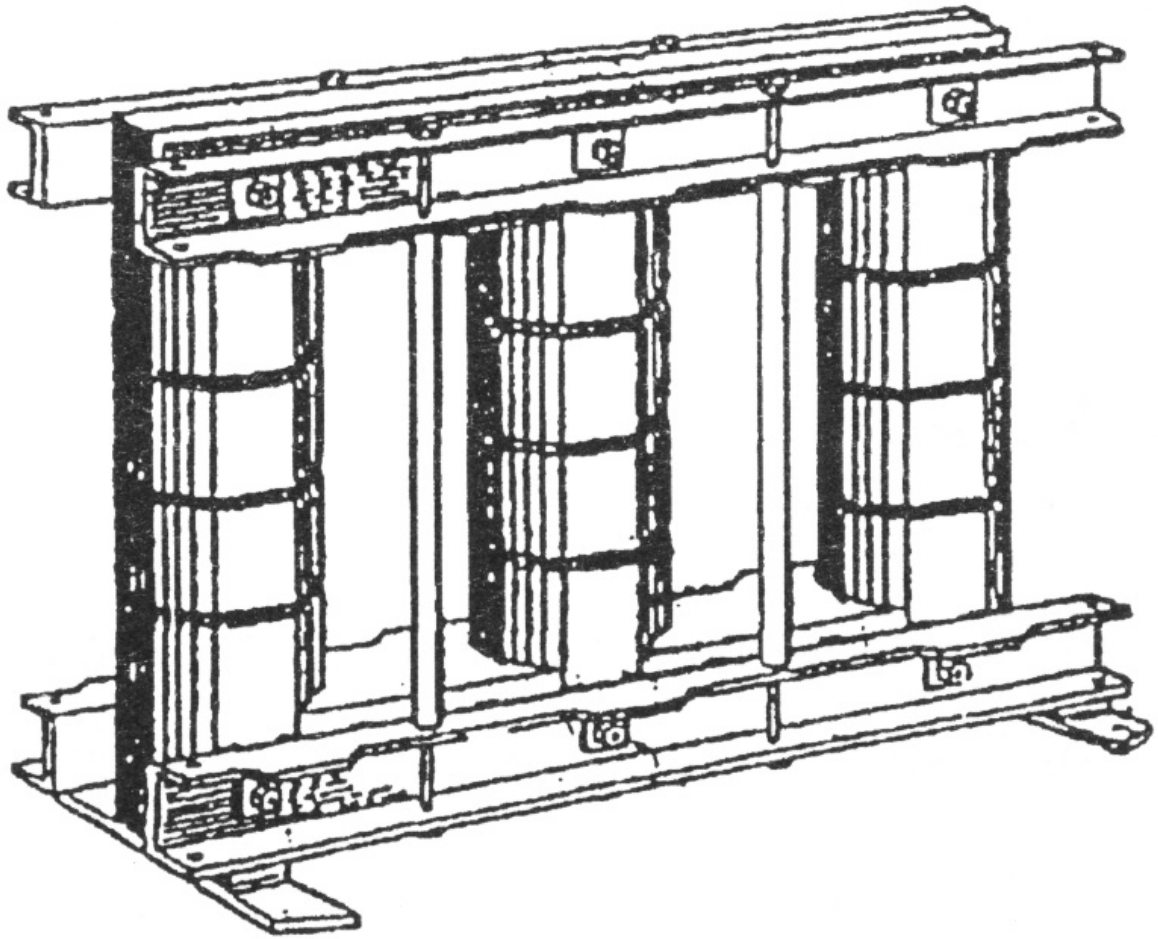
$$i_a + i_b + i_c = 0$$

$$\Phi_a + \Phi_b + \Phi_c = 0$$

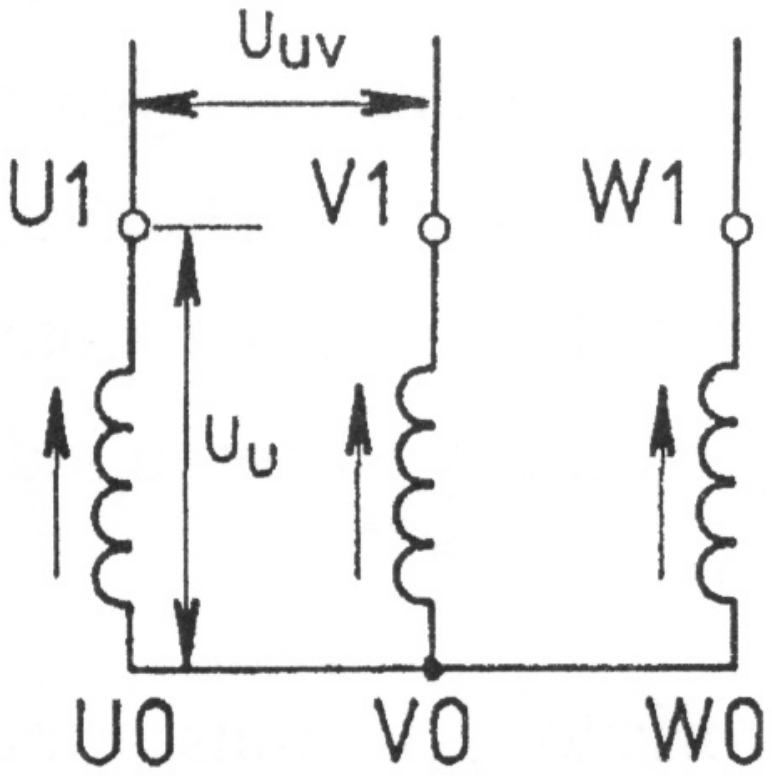
Three-Phase Transformer with Dependent Magnetic Circuits

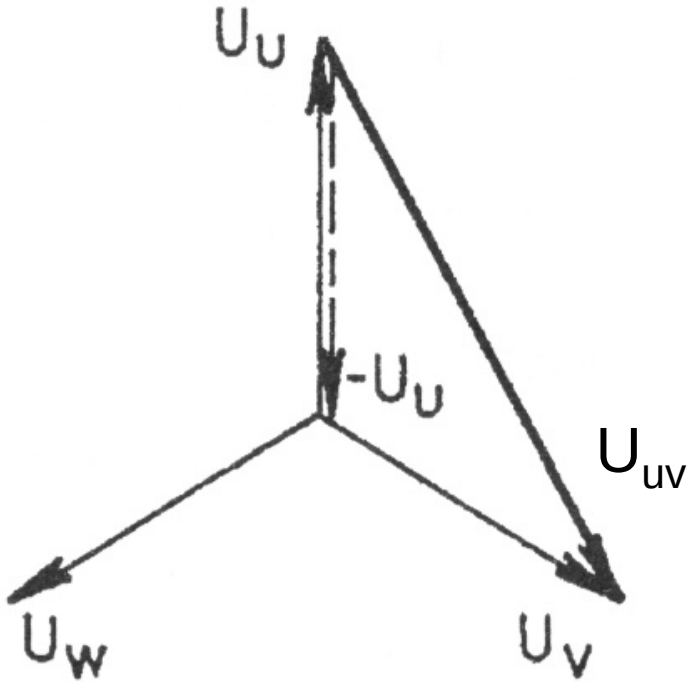


Magnetic Circuit of Three-Phase Transformer



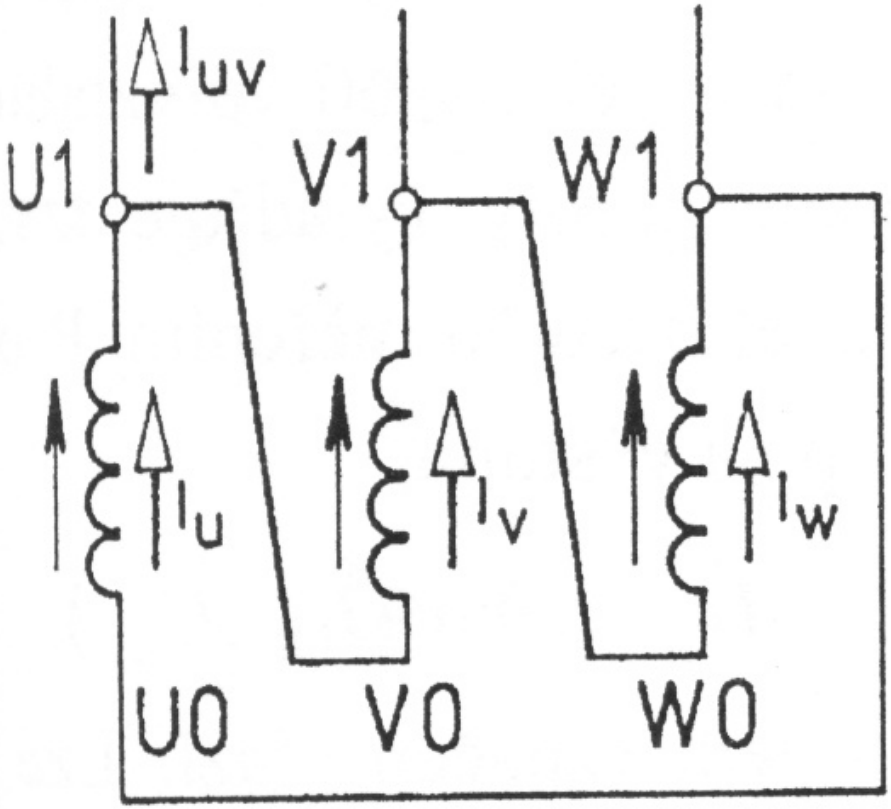
Star (wye) Connection

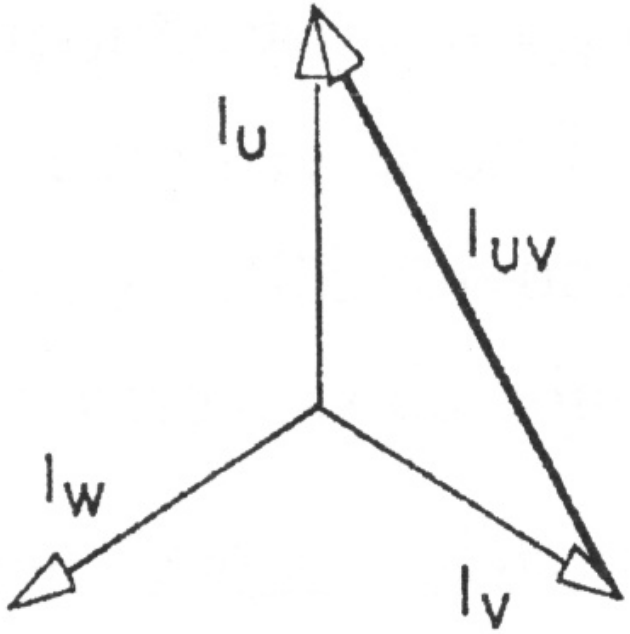




$$U_{UV} = \sqrt{3} \cdot U_U$$

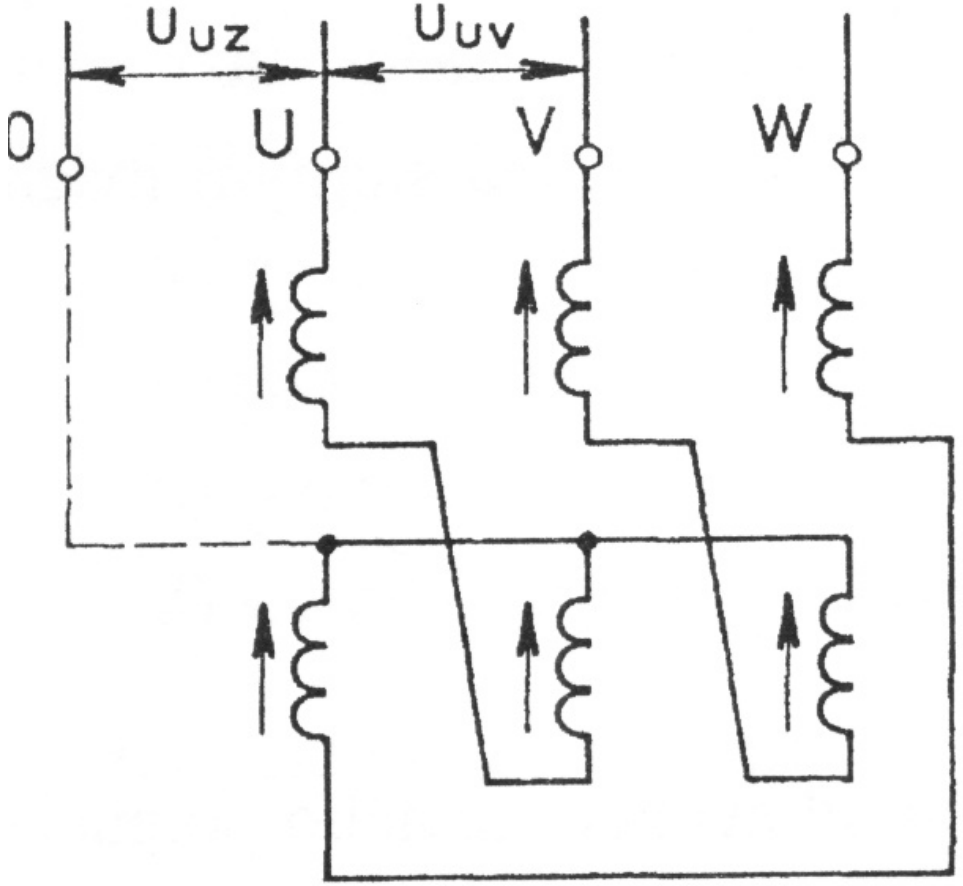
Triangle (delta) Connection

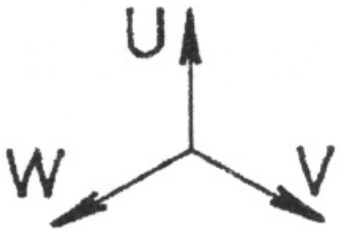


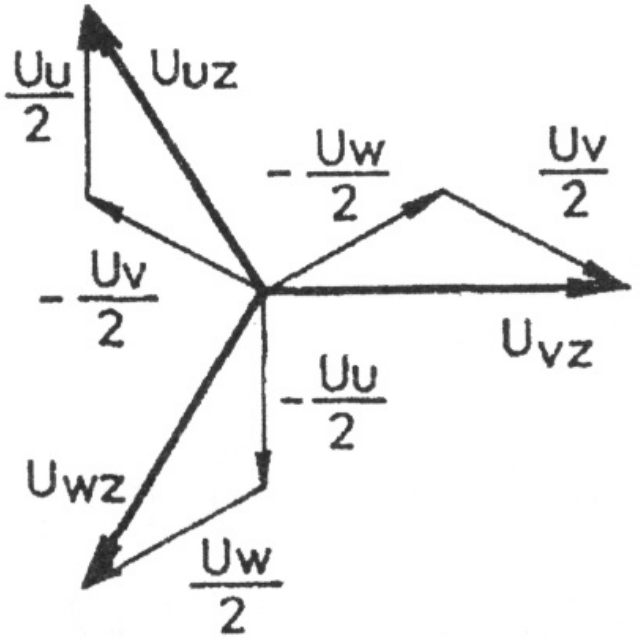


$$I_{UV} = \sqrt{3} \cdot I_U$$

Zigzag (Interconnected Star) Connection



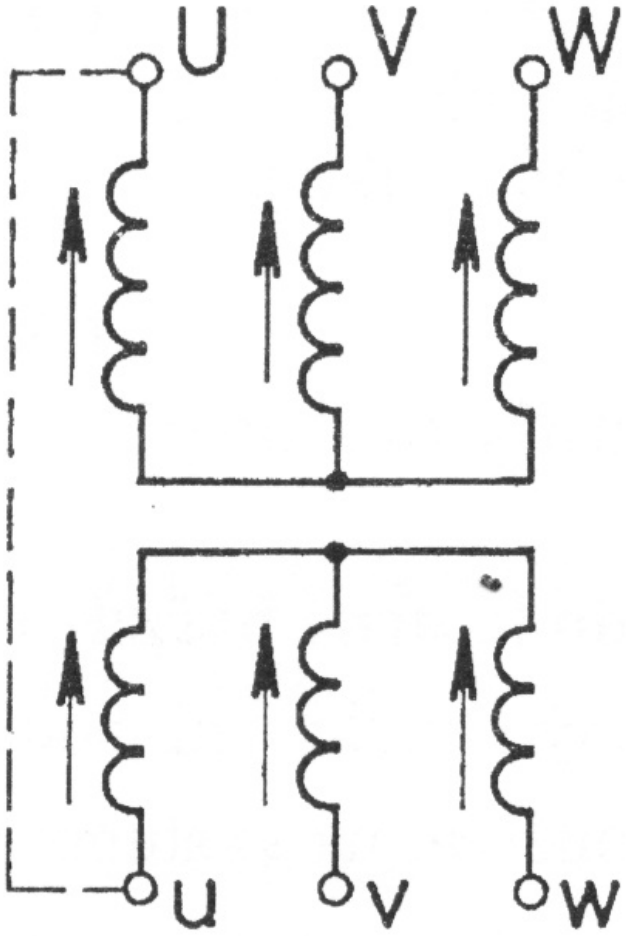




$$U_{UZ} = \left(\frac{U_U}{2} + \frac{U_V}{2} \right) \cos 30^\circ = \frac{\sqrt{3}}{2} \cdot U_U \cong 0{,}866 \cdot U_U$$

Vector group (hour angle)

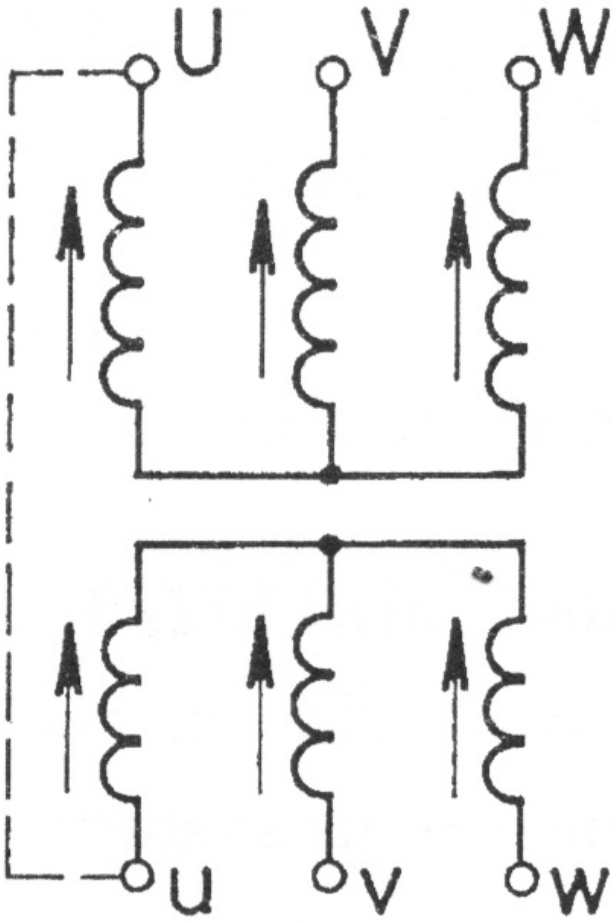
(indicates the windings configurations and the difference in phase angle between them)

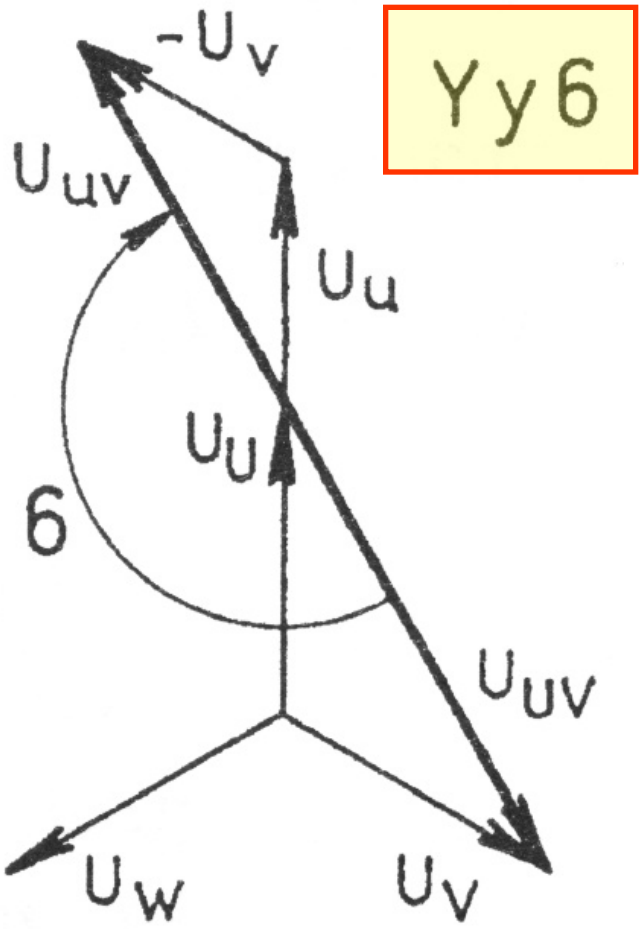


Y

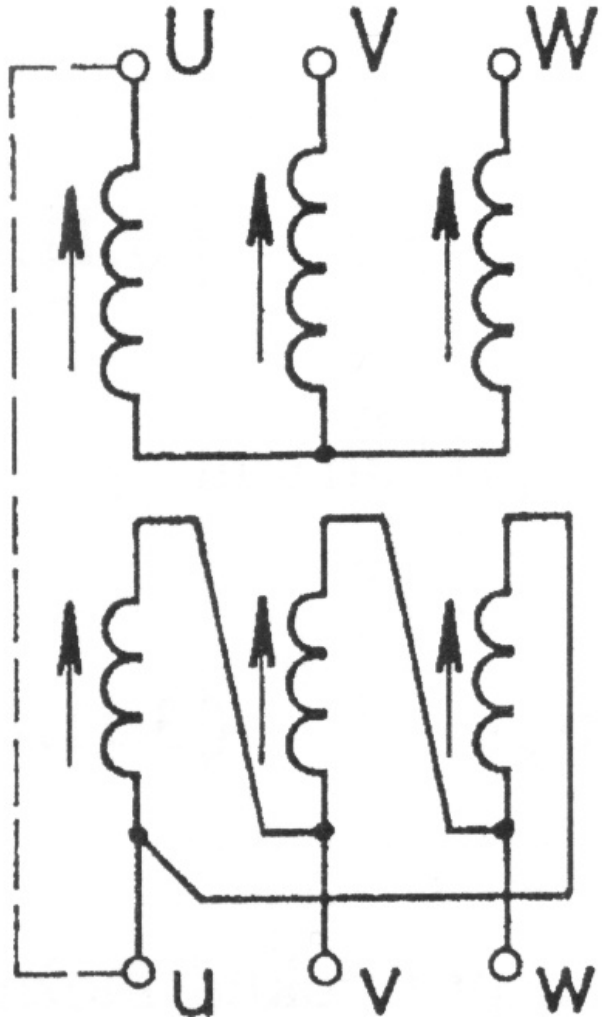
y

Vector group (hour angle)





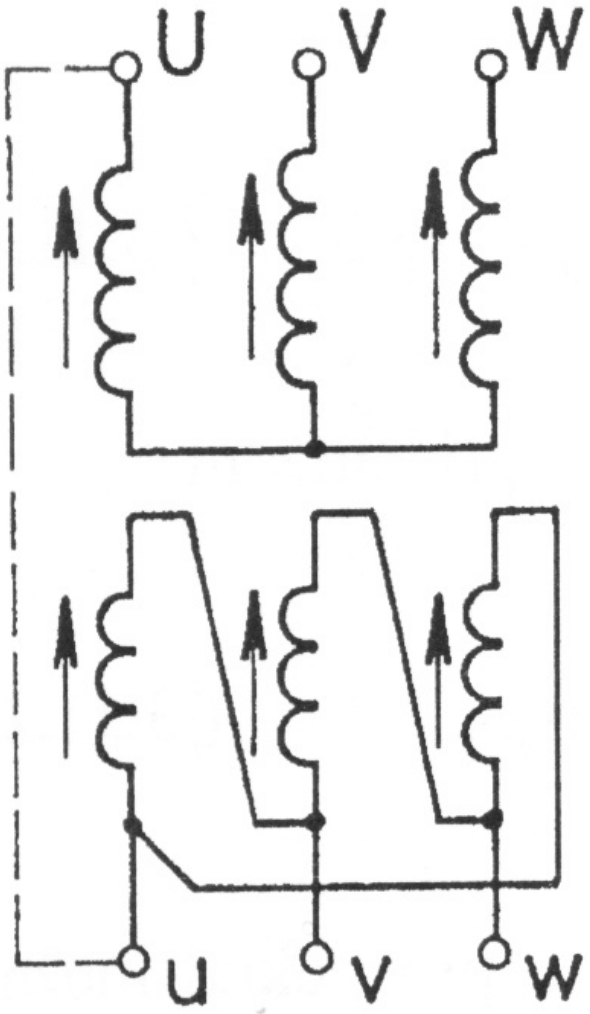
Vector group (hour angle)

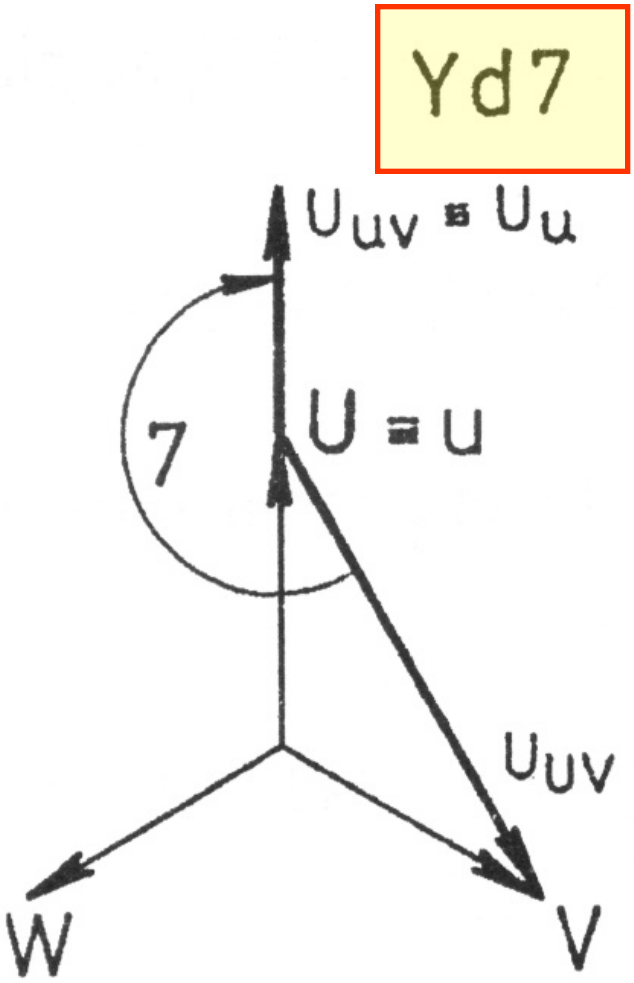


Y

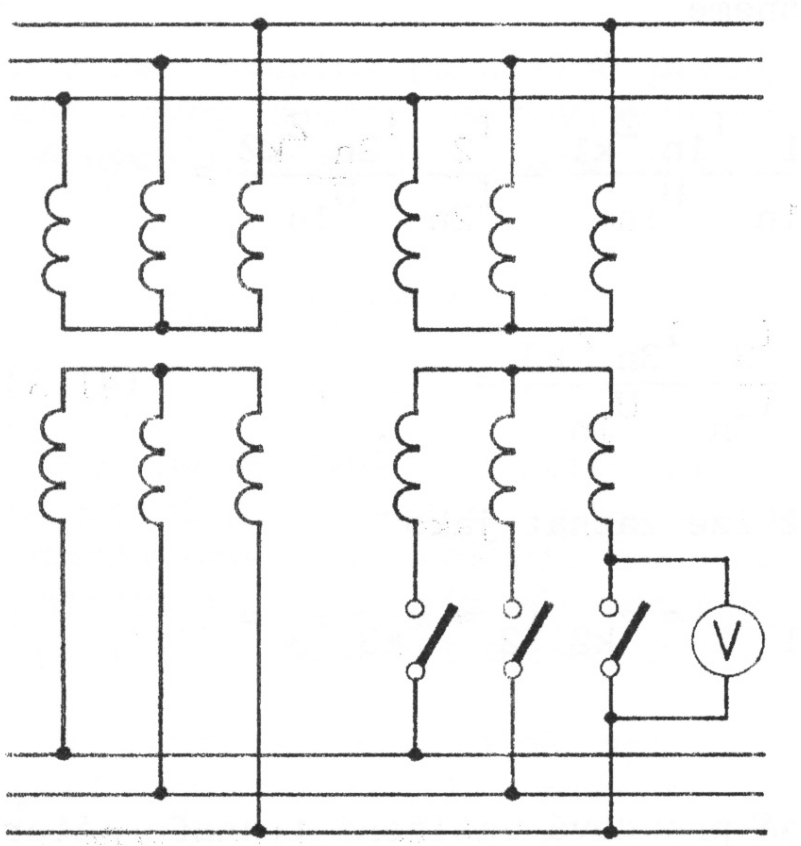
d

Vector group (hour angle)





Parallel operation of transformers



Requirement:

Balancing current $I_2 = 0$

$$\Rightarrow \quad \Delta \mathbf{U} = 0$$

Parallel Operation of Transformers

Requirements for parallel no-load operation:

1. Same nominal voltages
2. Same turns ratio
3. Same vector groups (hour angles)
4. Same voltage drop caused by no-load current

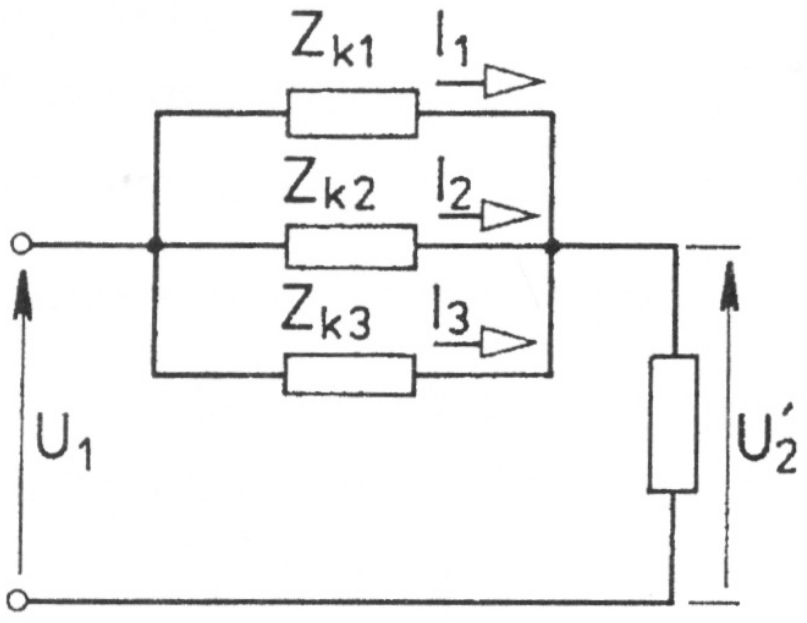
Parallel Operation of Transformers

Requirements for parallel operation of loaded transformers:

1. Same nominal voltages
2. Same turns ratio
3. Same vector groups (hour angles)
4. Same voltage drop caused by no-load current
5. Same voltage drop when loaded

$$Z_{k1} I_1 = Z_{k2} I_2$$

Parallel Operation of 3 Transformers



5. Same voltage drop when loaded

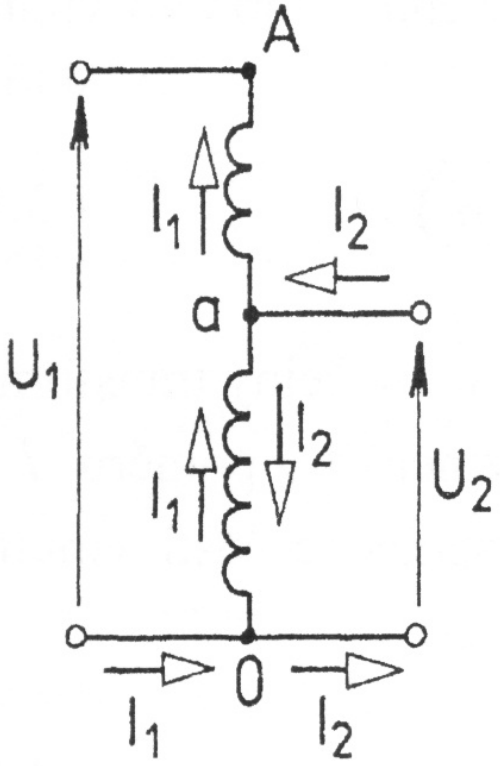
$$Z_{k1}I_1 = Z_{k2}I_2 = Z_{k3}I_3$$

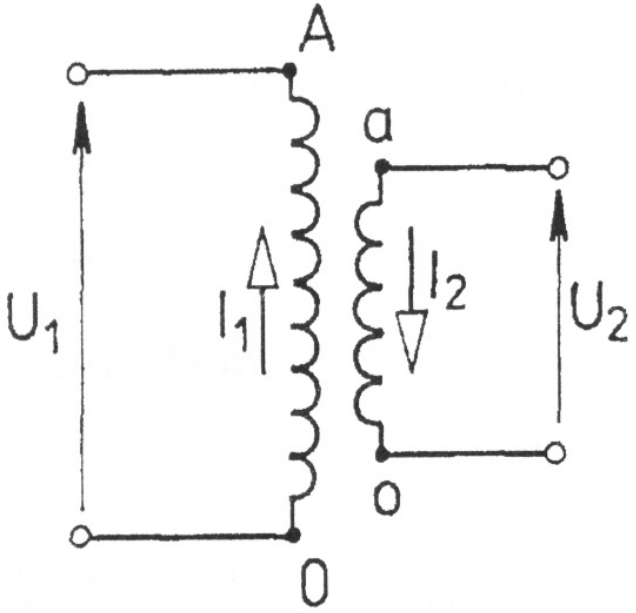
$$z_{k1}\dot{i}_1 = z_{k2}\dot{i}_2 = z_{k3}\dot{i}_3$$

$$z_{k1} = z_{k2} = z_{k3}$$

$$u_{k1} = u_{k2} = u_{k3}$$

Autotransformer



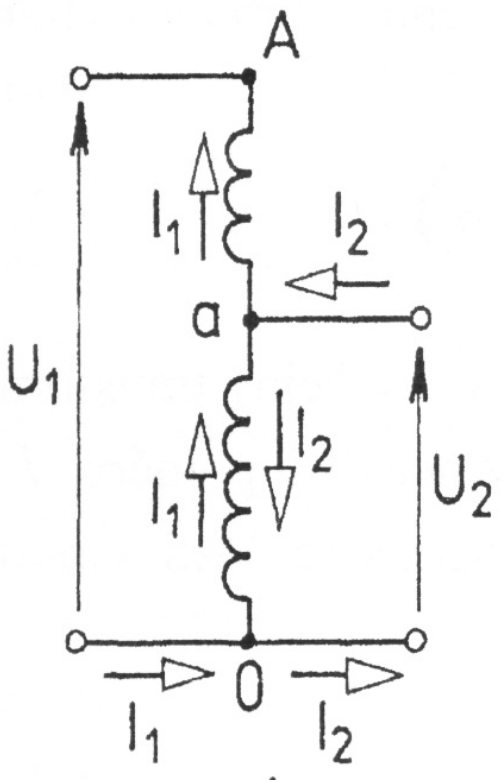


Part of the winding common for primary and secondary

→ lower weight, costs, losses, I_0 .

Input – output galvanically connected.

Autotransformer



For $p = \frac{U_1}{U_2} > 1$

Current I_1 flows only through Aa part of the winding

Lower current flows through common part of the winding

$$\hat{I} = \hat{I}_1 + \hat{I}_2$$

For normal transformer:

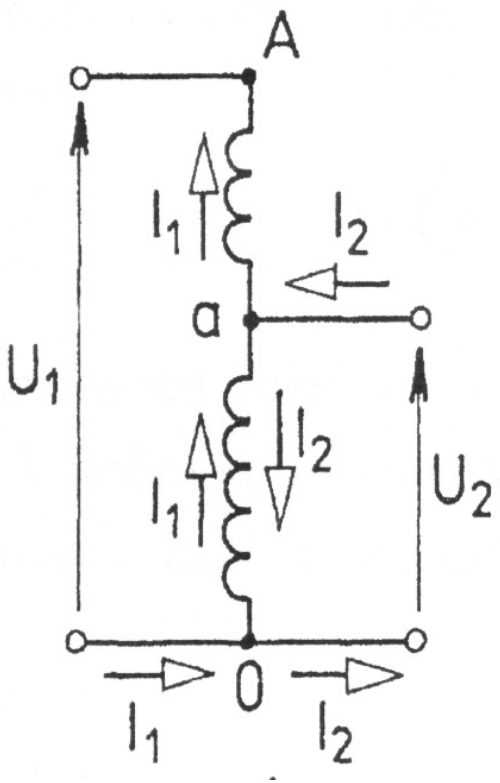
$$N_1\hat{I}_1 + N_2\hat{I}_2 = \hat{F}_\mu \cong 0$$

$$\hat{I}_1 + \frac{\hat{I}_2}{p} \cong 0$$

Current in the common part of the autotransformer winding:

$$I = I_1(1-p) = I_2(1-1/p)$$

Autotransformer



For $p = \frac{U_1}{U_2} > 1$

Power to secondary side is transmitted in form of:

$$S_1 = U_1 \cdot I_1 \approx U_2 \cdot I_2$$

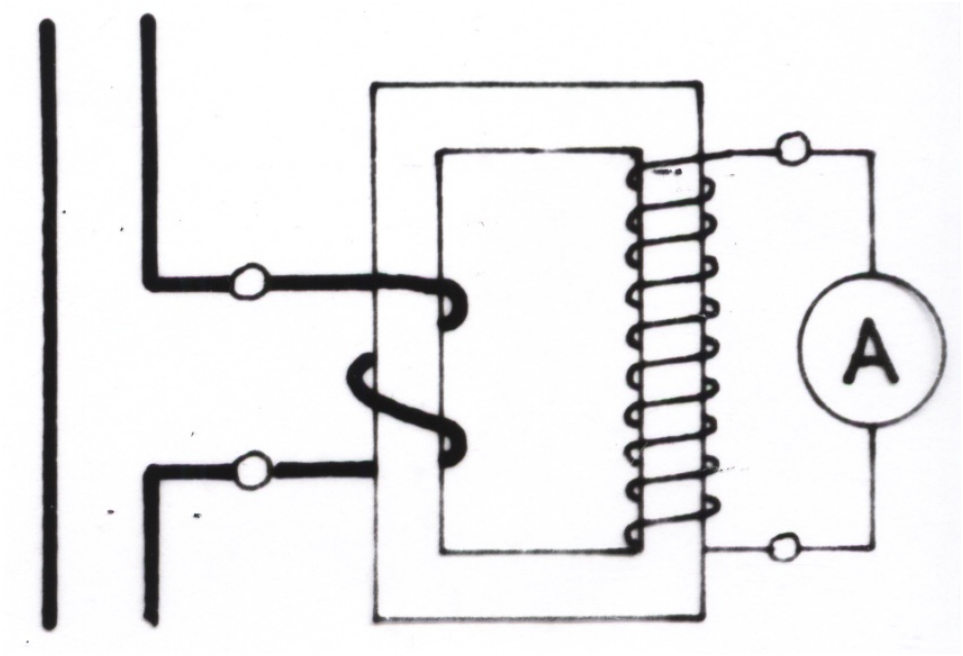
Typical power:
(magnetic transmission)

$$S_{12} = (U_1 - U_2) I_1 = U_1 I_1 - U_2 I_1 = S_1 (1 - 1/p)$$

Electric power:
(galvanic transmission)

$$S_{el} = S_1 - S_{12} = S_1 \cdot 1/p$$

Current Instrument Transformer

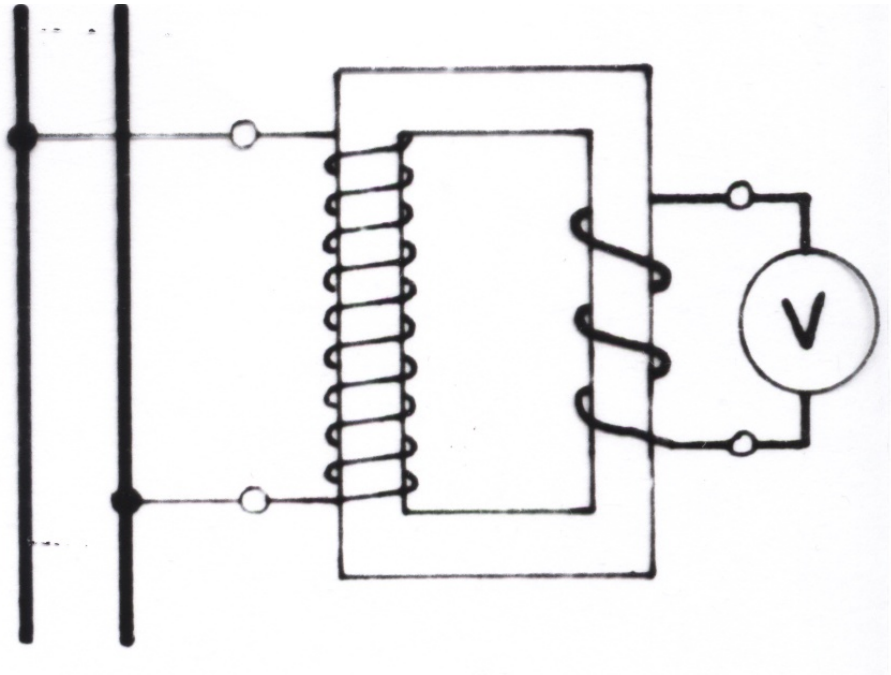


Load: ammeter ~ short-circuit

Range: 5 A

Current instrument transformer must not be disconnected on output side!

Voltage Instrument Transformer

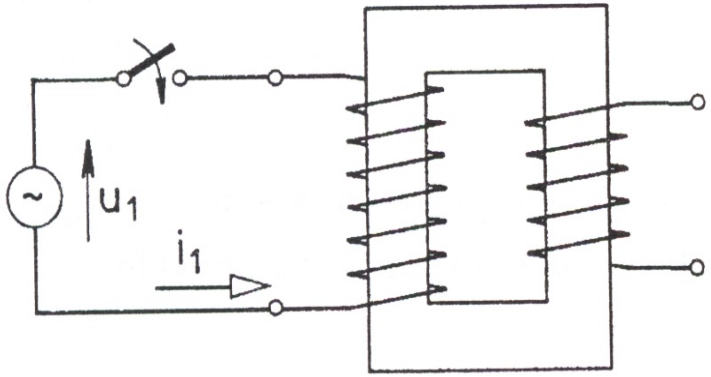


Load: voltmeter ~ no-load

Range: 100 V

Voltage instrument transformer must not be short-circuited on output side!

Connection of Transformer in No-Load Operation to Grid



$$u_1 = R_1 \cdot i_1 + \frac{d\Psi}{dt} = R_1 \cdot i_1 + N_1 \cdot \frac{d\varphi_{11}}{dt}$$

Supposing:

$$i_1 = \frac{N_1 \cdot \varphi_{11}}{L_{11}}, \quad u_1 = U_{1\max} \cdot \sin(\omega t + \alpha)$$

then:

$$\frac{u_1}{N_1} = \frac{R_1}{L_{11}} \cdot \varphi_{11} + \frac{d\varphi_{11}}{dt} = \frac{U_{1\max}}{N_1} \cdot \sin(\omega t + \alpha)$$

Solution:

$$\varphi_{11} = \varphi_{11\max}\left[-\cos(\omega t - \alpha) + e^{-\frac{R_1}{L_{11}}t}\cos\alpha\right] \pm \varphi_{zb}e^{-\frac{R_1}{L_{11}}t}$$

Steady magnetic flux

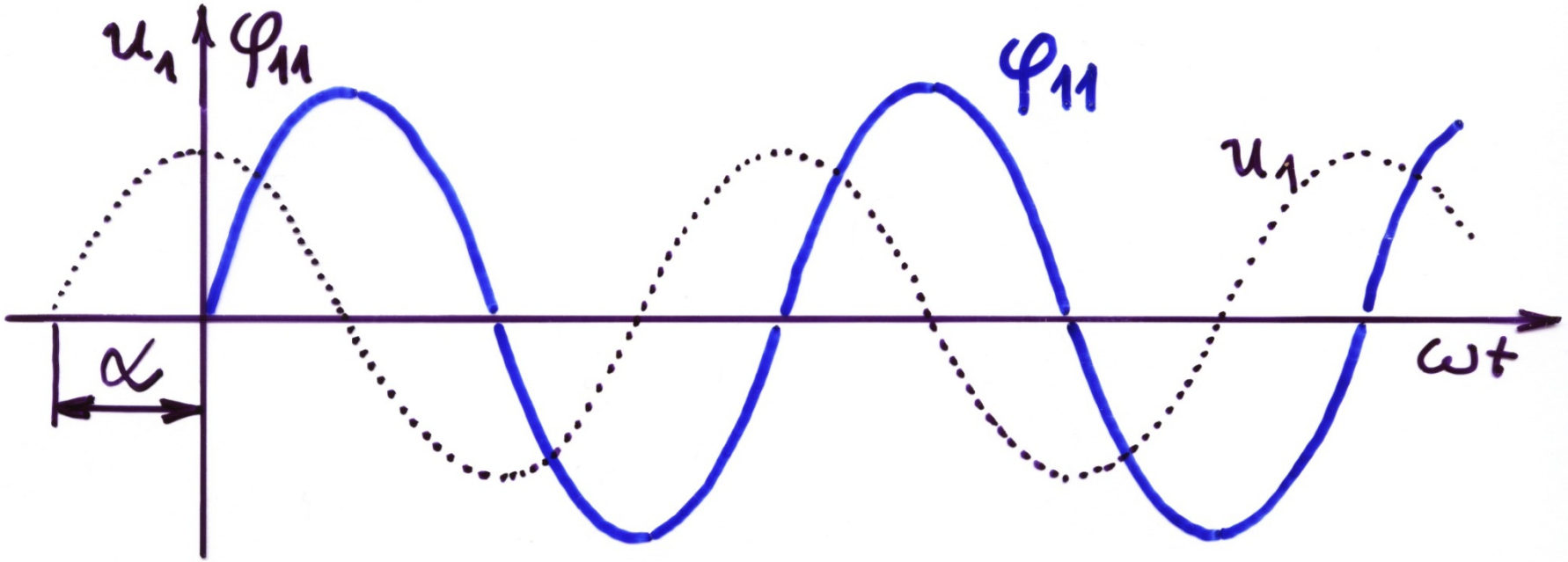
DC component of magnetic flux

Remanent flux

Connection of Transformer in No-Load Operation to Grid

1) $\alpha = \frac{\pi}{2}; \quad \varphi_{zb} = 0$

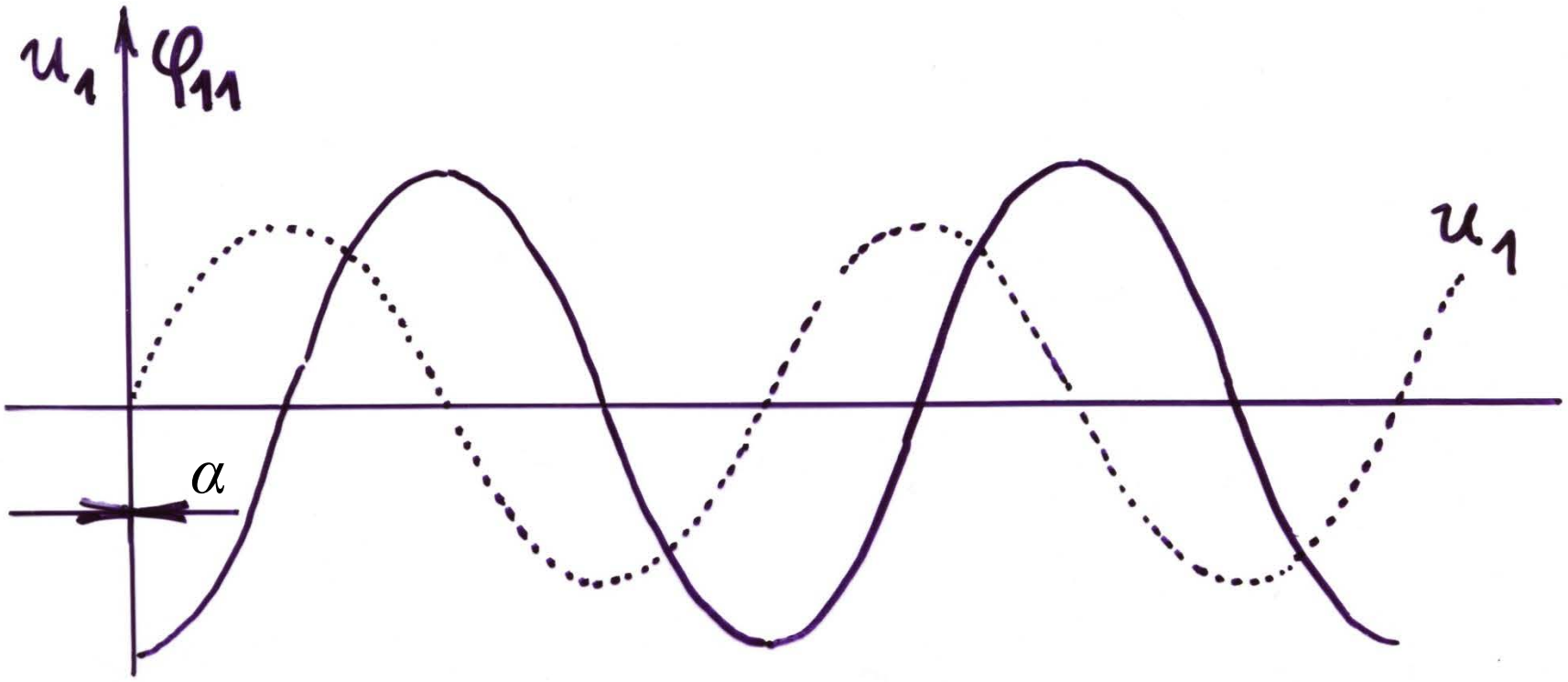
$$\varphi_{11} = -\varphi_{11\max} \cos\left(\omega t + \frac{\pi}{2}\right) = \varphi_{11\max} \sin \omega t$$



Connection of Transformer in No-Load Operation to Grid

2) $\alpha = 0\,; \quad \varphi_{zb} = 0$

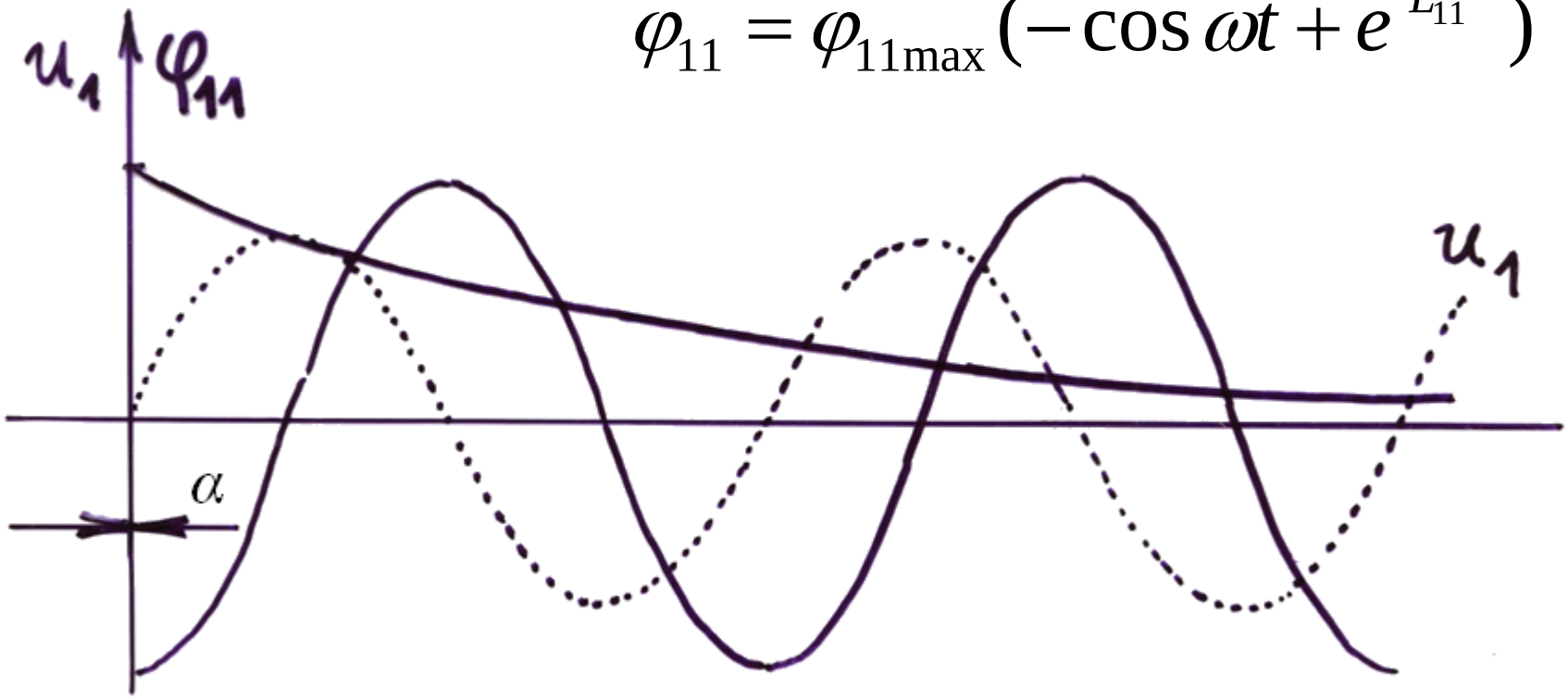
$$\varphi_{11} = \varphi_{11\max}\left(-\cos\omega t + e^{\frac{-R_1}{L_{11}}t}\right)$$



Connection of Transformer in No-Load Operation to Grid

2) $\alpha = 0\,; \quad \varphi_{zb} = 0$

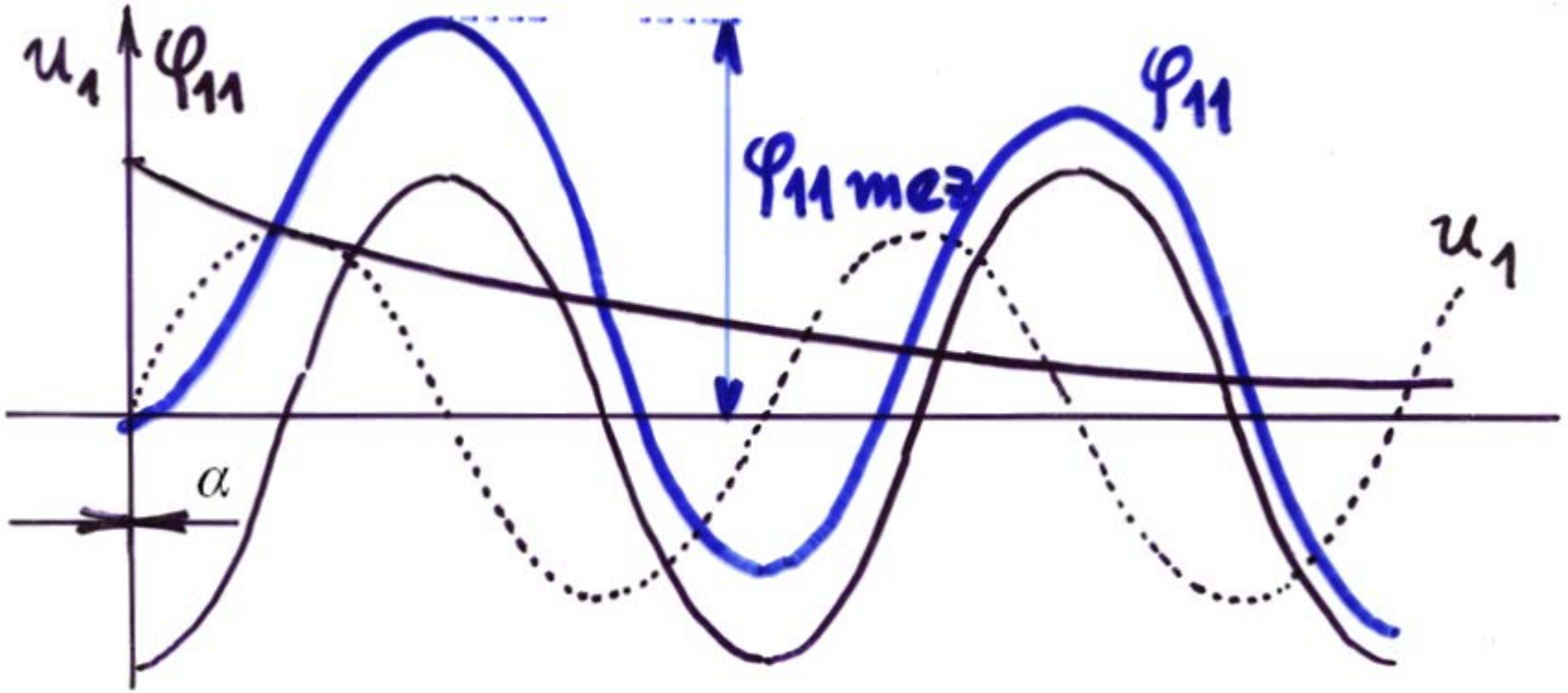
$$\varphi_{11} = \varphi_{11\max}\left(-\cos\omega t + e^{\frac{-R_1}{L_{11}}t}\right)$$



Connection of Transformer in No-Load Operation to Grid

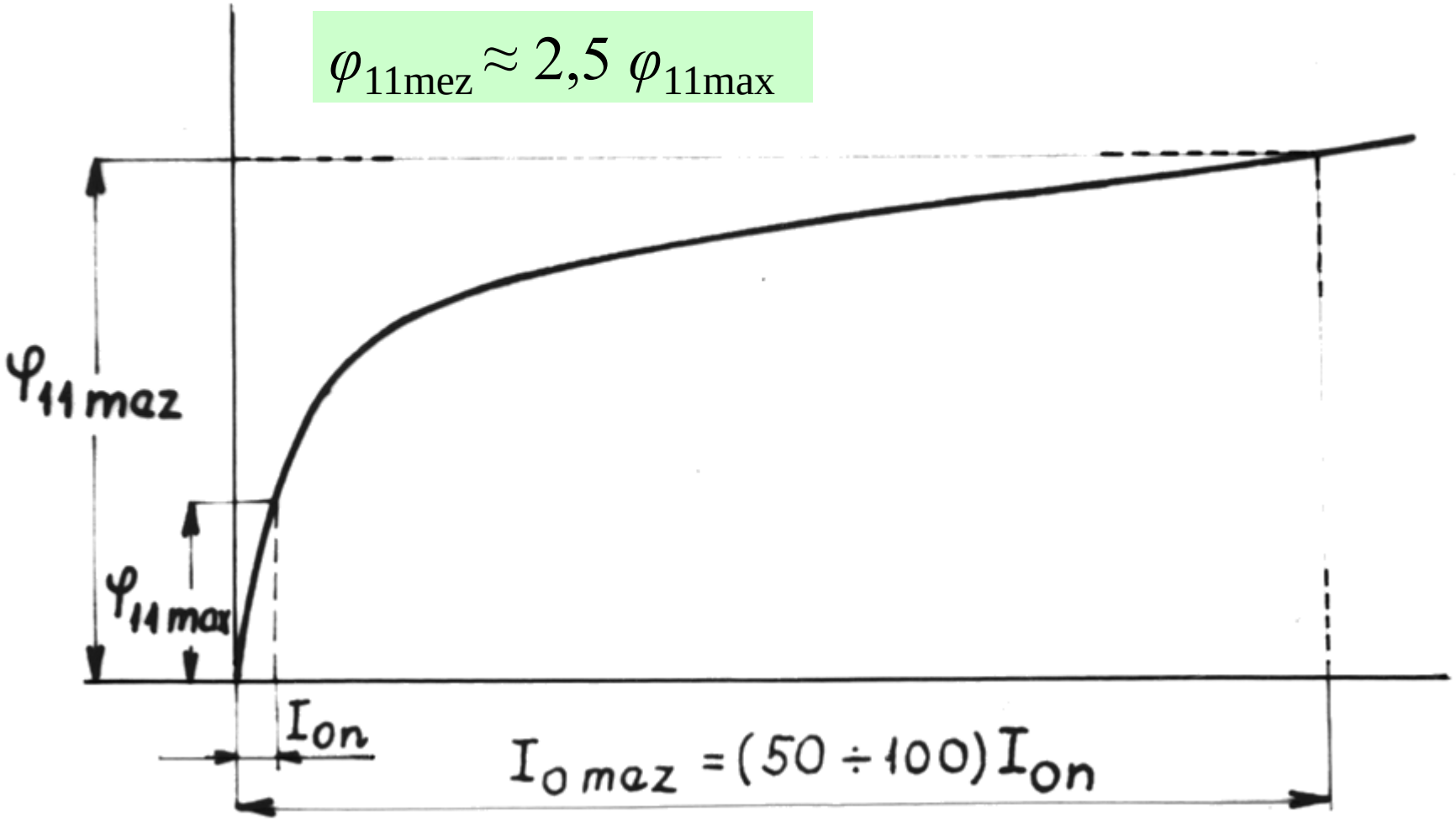
2) $\alpha = 0\,; \quad \varphi_{zb} = 0$

$$\varphi_{11} = \varphi_{11\max}\left(-\cos\omega t + e^{\frac{-R_1}{L_{11}}t}\right)$$



Connection of Transformer in No-Load Operation to Grid

$$\varphi_{11mez} \approx 2{,}5\ \varphi_{11max}$$



Highest current by connection.

Short Circuit of Transformer in Steady-State No-Load Operation

Fundamental voltage equation:

$$u_1 = R_k i_1 + L_k \frac{di_1}{dt} = U_{1\max} \sin(\omega t + \alpha)$$

Solution for $i_1 = 0$ at $t = 0$

$$i_1 = \sqrt{2} \cdot I_k \left[\sin(\omega t + \alpha - \varphi_k) - \sin(\alpha - \varphi_k) e^{-\frac{R_k}{L_k} t} \right]$$

Angle α is determined by instant of short-circuit.

Extreme cases:

1) $\alpha = 0$

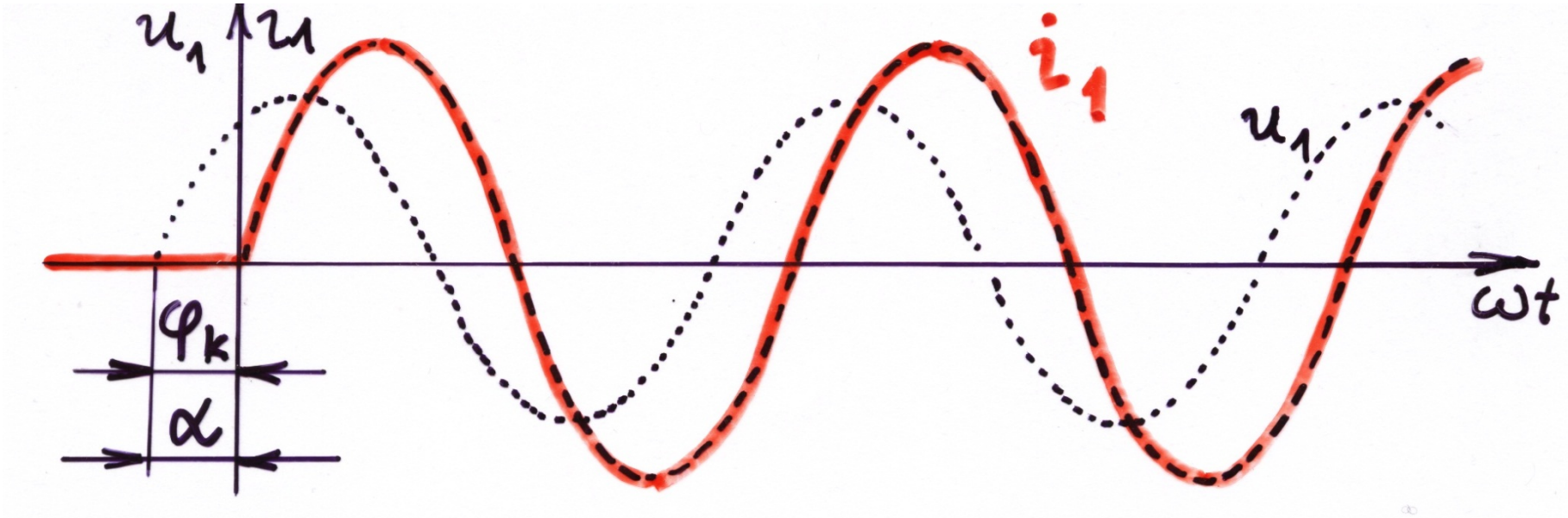
2) $\alpha = \varphi_k + \pi/2$

Extreme Cases of Short-Circuit

$$\alpha = \varphi_k$$

Solution to this is the steady short-circuit current

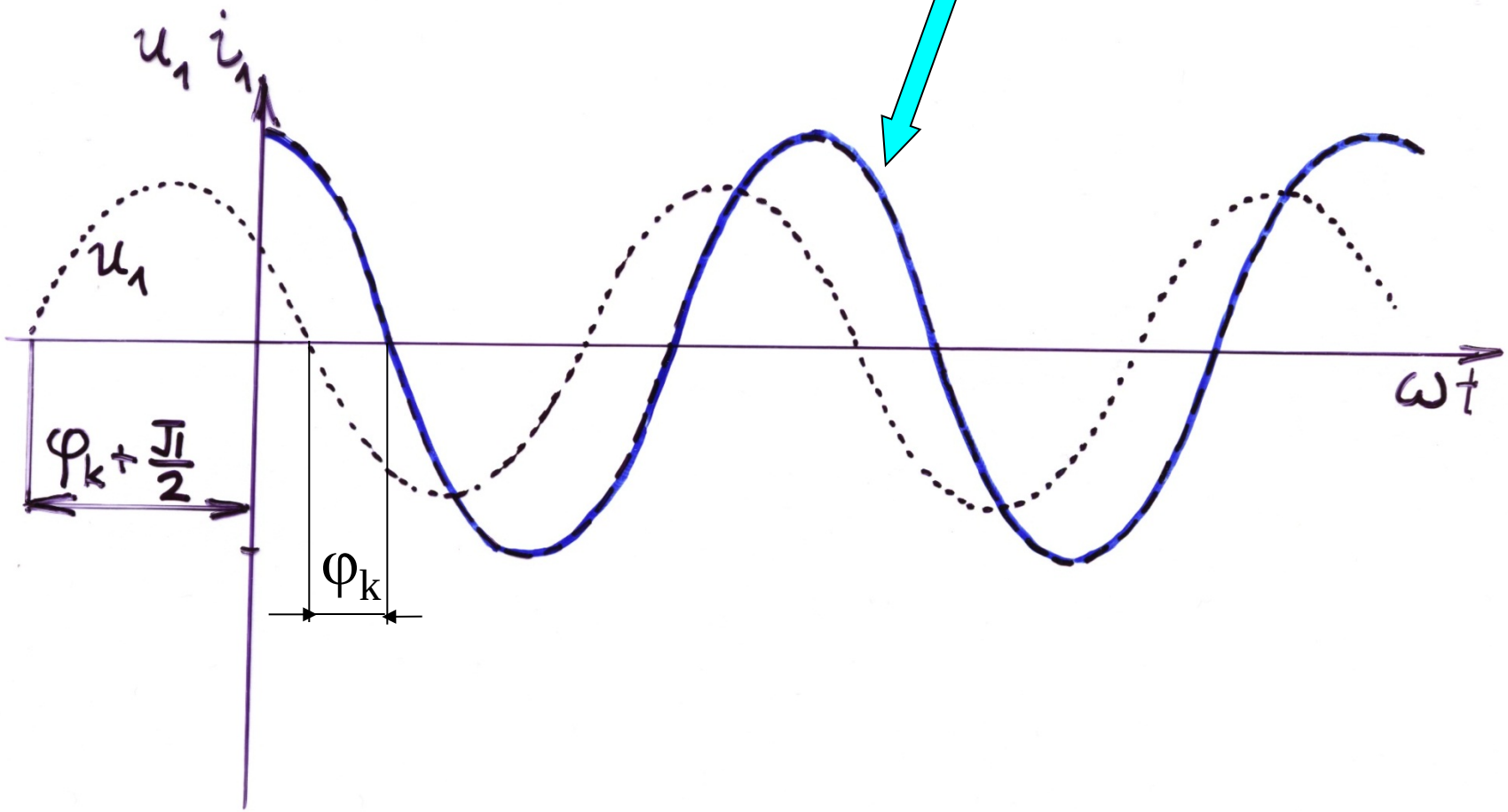
$$i_1 = \sqrt{2} \cdot I_{1k} \sin \omega t$$



Extreme Cases of Short-Circuit

$$\alpha = \varphi_k + \frac{\pi}{2}$$

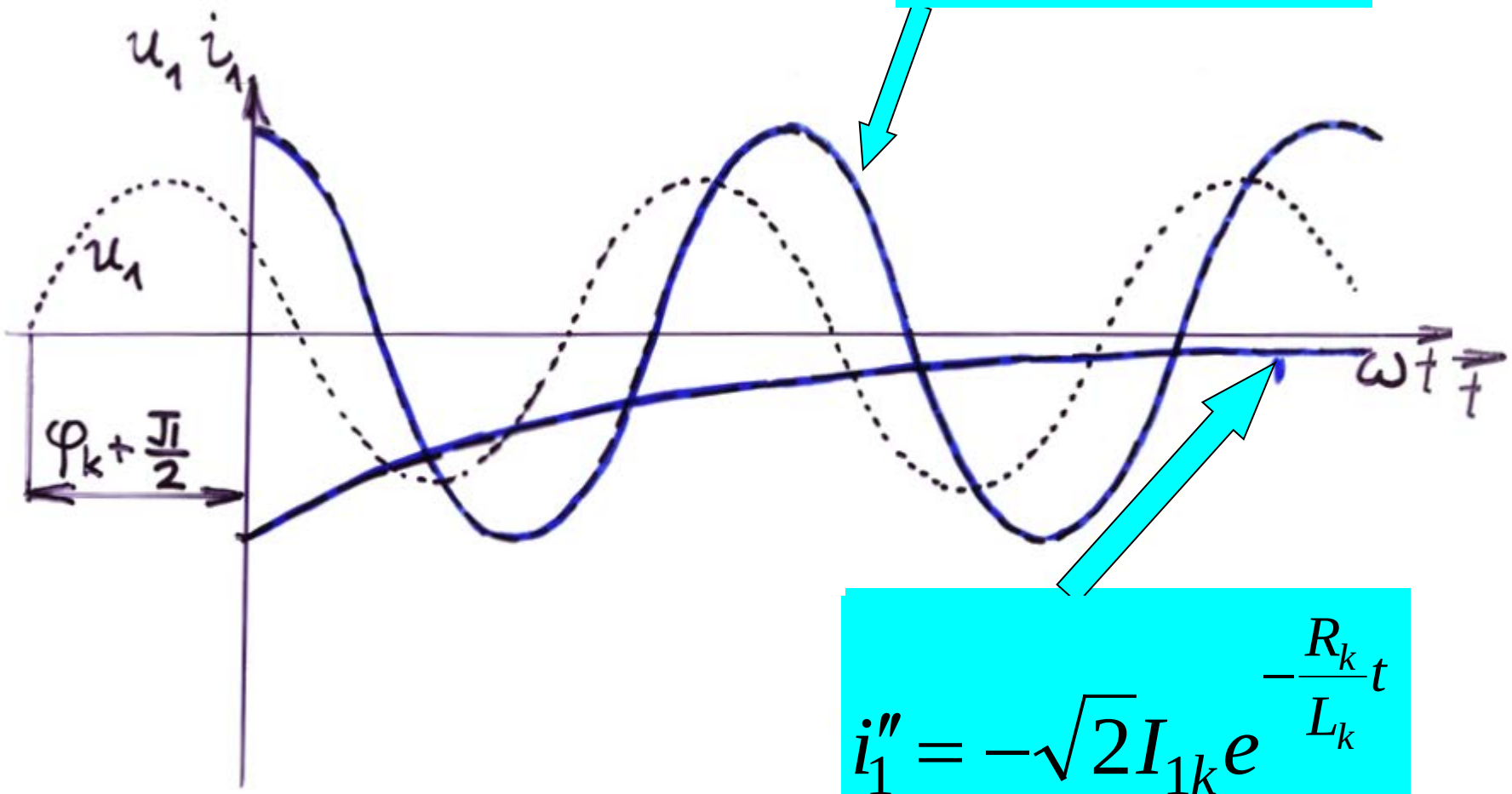
$$i_1' = \sqrt{2} \cdot I_{1k} . \cos \omega t$$



Extreme Cases of Short-Circuit

$$\alpha = \varphi_k + \frac{\pi}{2}$$

$$i_1' = \sqrt{2} \cdot I_{1k} \cdot \cos \omega t$$



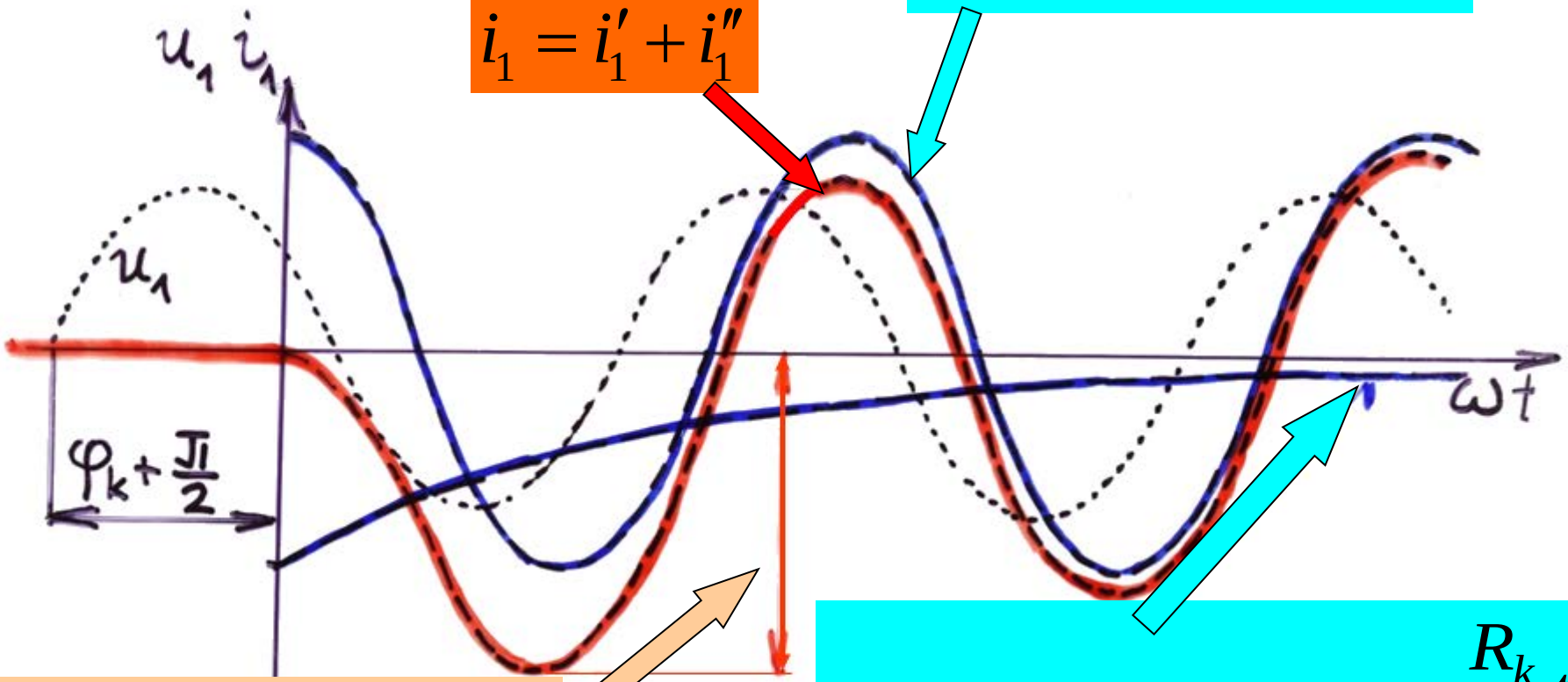
$$i_1'' = -\sqrt{2} I_{1k} e^{-\frac{R_k}{L_k} t}$$

Extreme Cases of Short-Circuit

$$\alpha = \varphi_k + \frac{\pi}{2}$$

$$i_1 = i_1' + i_1''$$

$$i_1' = \sqrt{2} \cdot I_{1k} . \cos \omega t$$



$$I_{1mez} = -\sqrt{2} \frac{U_1}{Z_k} \left(1 + e^{-\frac{R_k}{X_k}\pi} \right)$$

$$i_1'' = -\sqrt{2} . I_{1k} . e^{-\frac{R_k}{L_k} t}$$

Limitation of Short-Circuit Current

$$I_{1\lim} = -\sqrt{2}\frac{U_1}{Z_k}\left(1 + e^{-\frac{R_k}{X_k}\pi}\right)$$

Big transformers:

$$I_{1\lim} = (1,7 \div 1,8)\sqrt{2} \cdot I_{1k} \quad tj.(30 \div 40) \cdot I_{1N}$$

Small transformers:

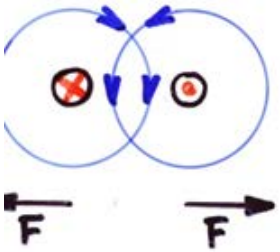
$$I_{1\lim} = (1,2 \div 1,3)\sqrt{2} \cdot I_{1k} \quad tj.(15 \div 20) \cdot I_{1N}$$

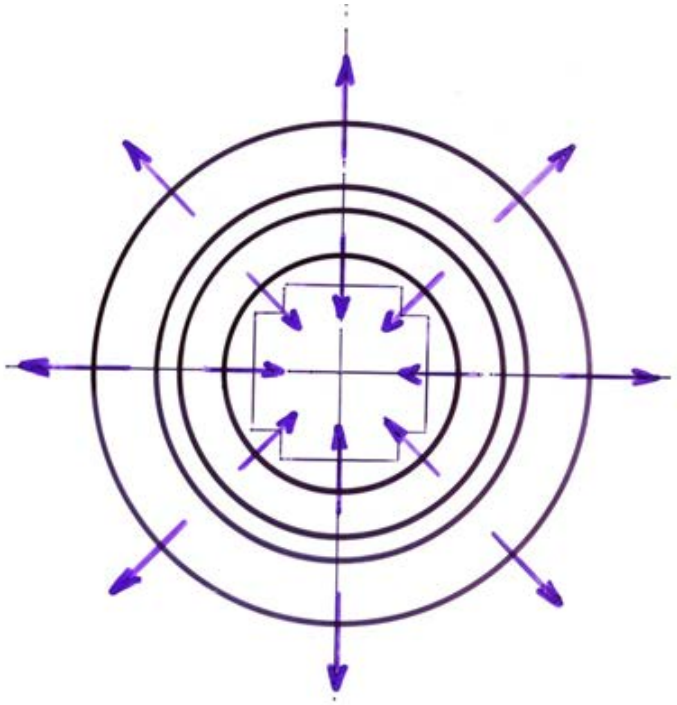
Mechanical Stress of Transformers

Forces ~ $I_1 \cdot I_2$ or more precisely I^2

During short-circuit → SHORT-CIRCUIT FORCES

Ⓐ RADIAL

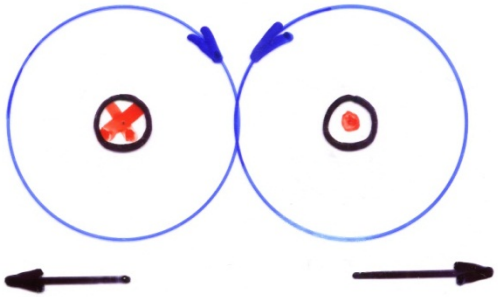


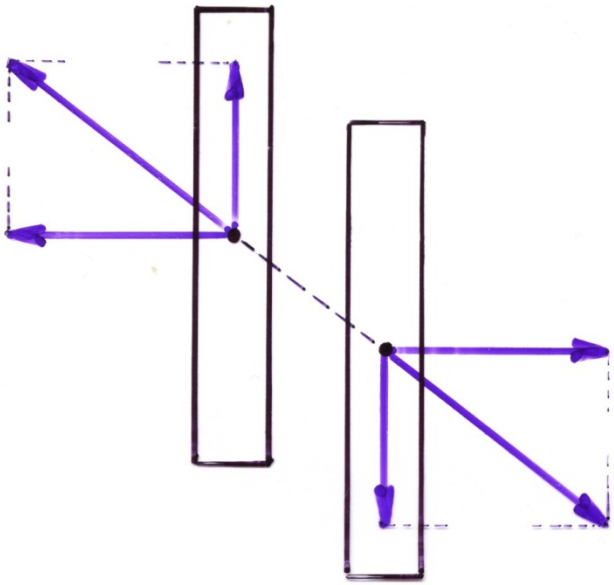


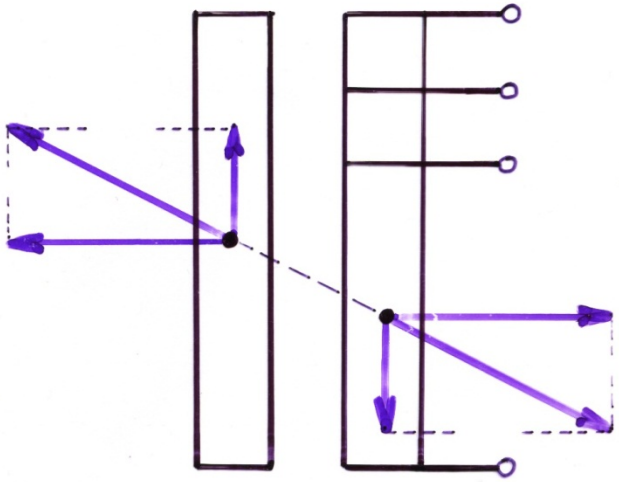
Mechanical Stress of Transformers

- AXIAL

BETWEEN THE WINDINGS

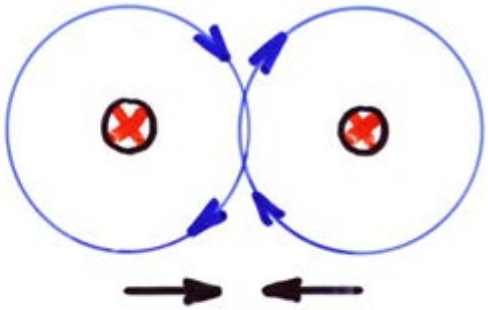


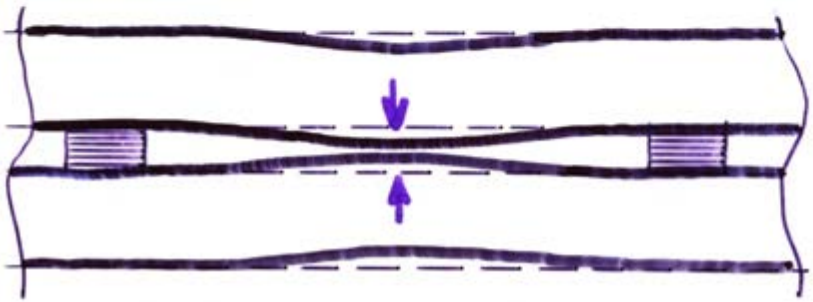




Mechanical Stress of Transformers

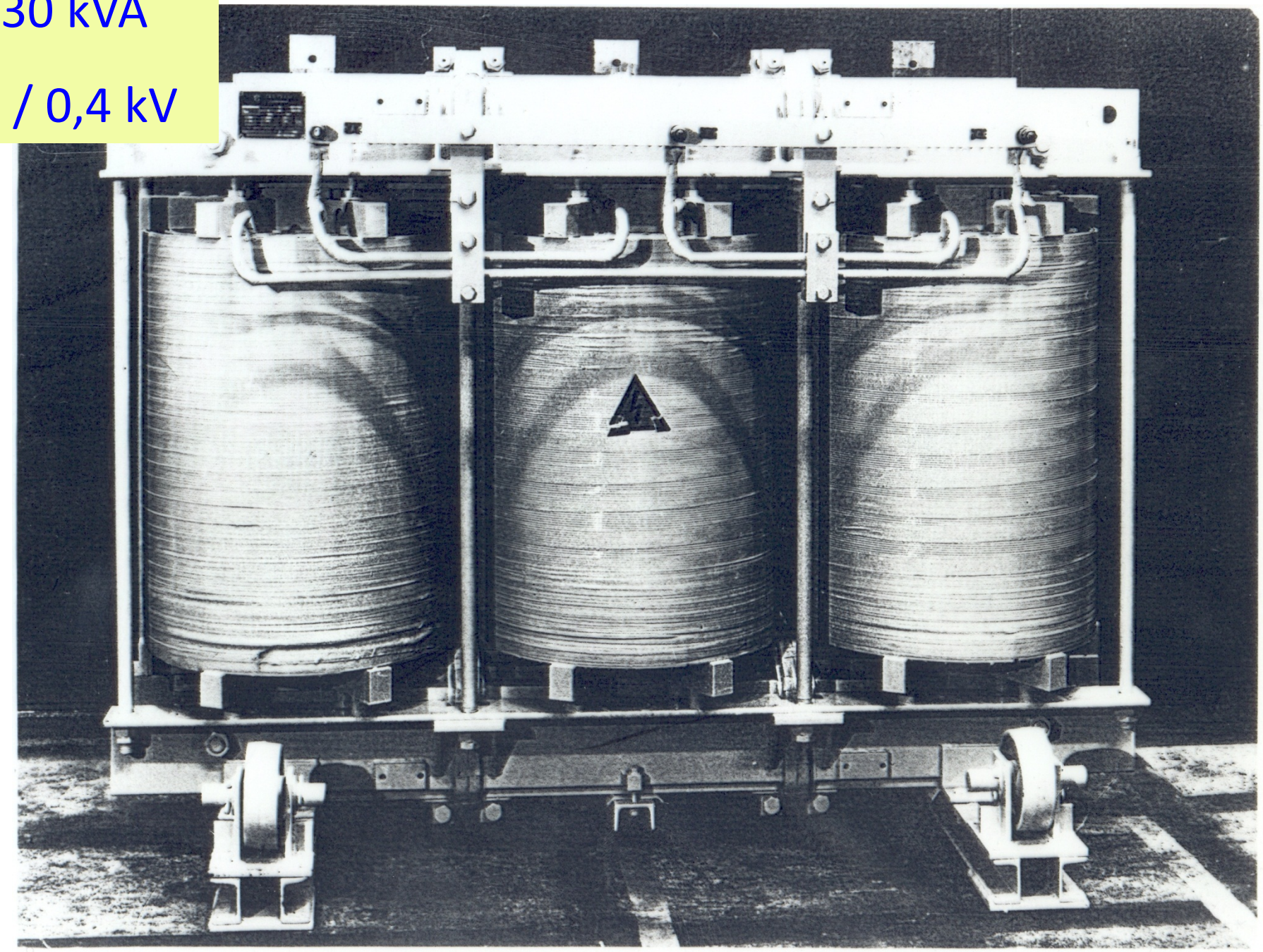
INSIDE THE WINDING

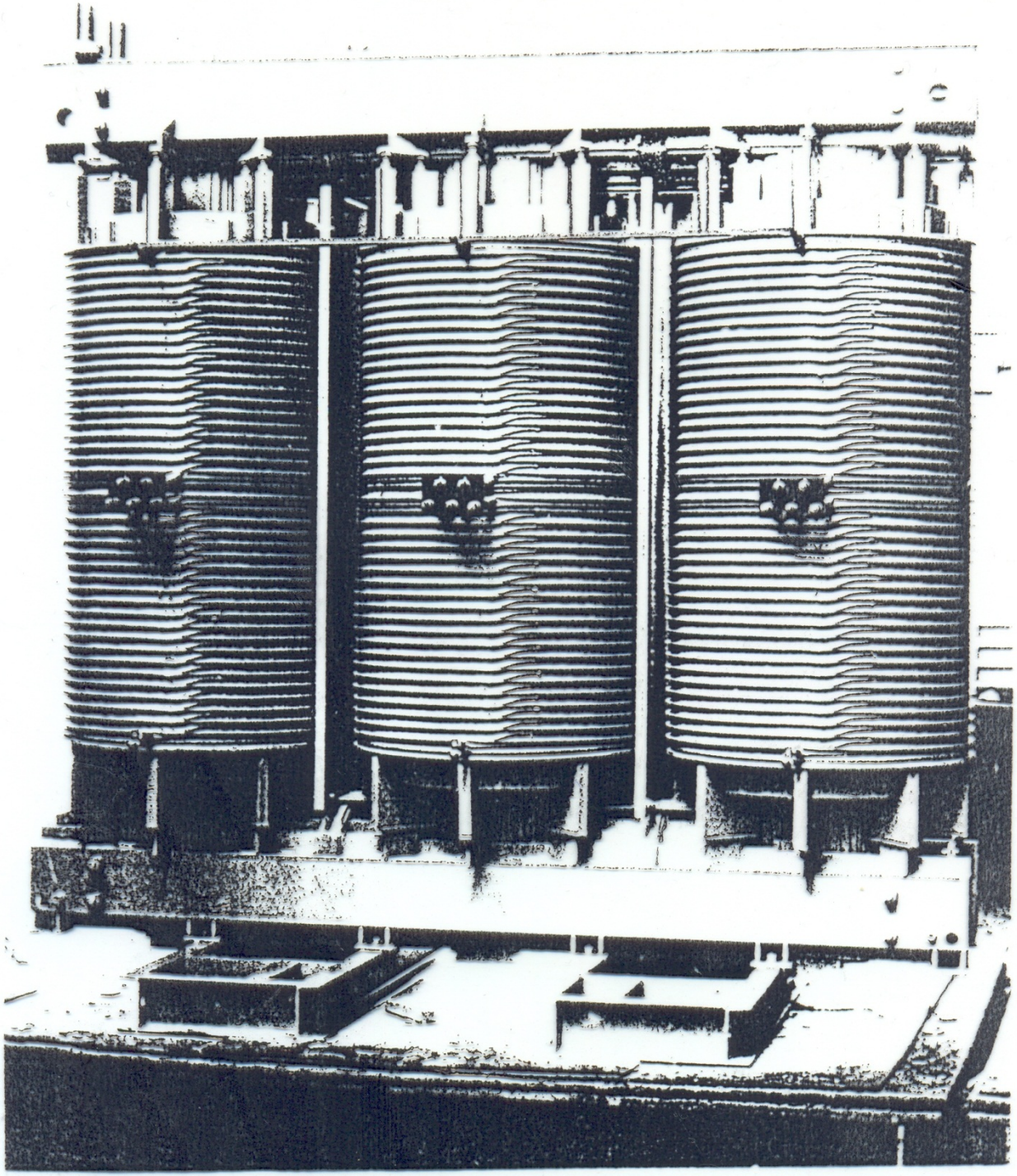




630 kVA

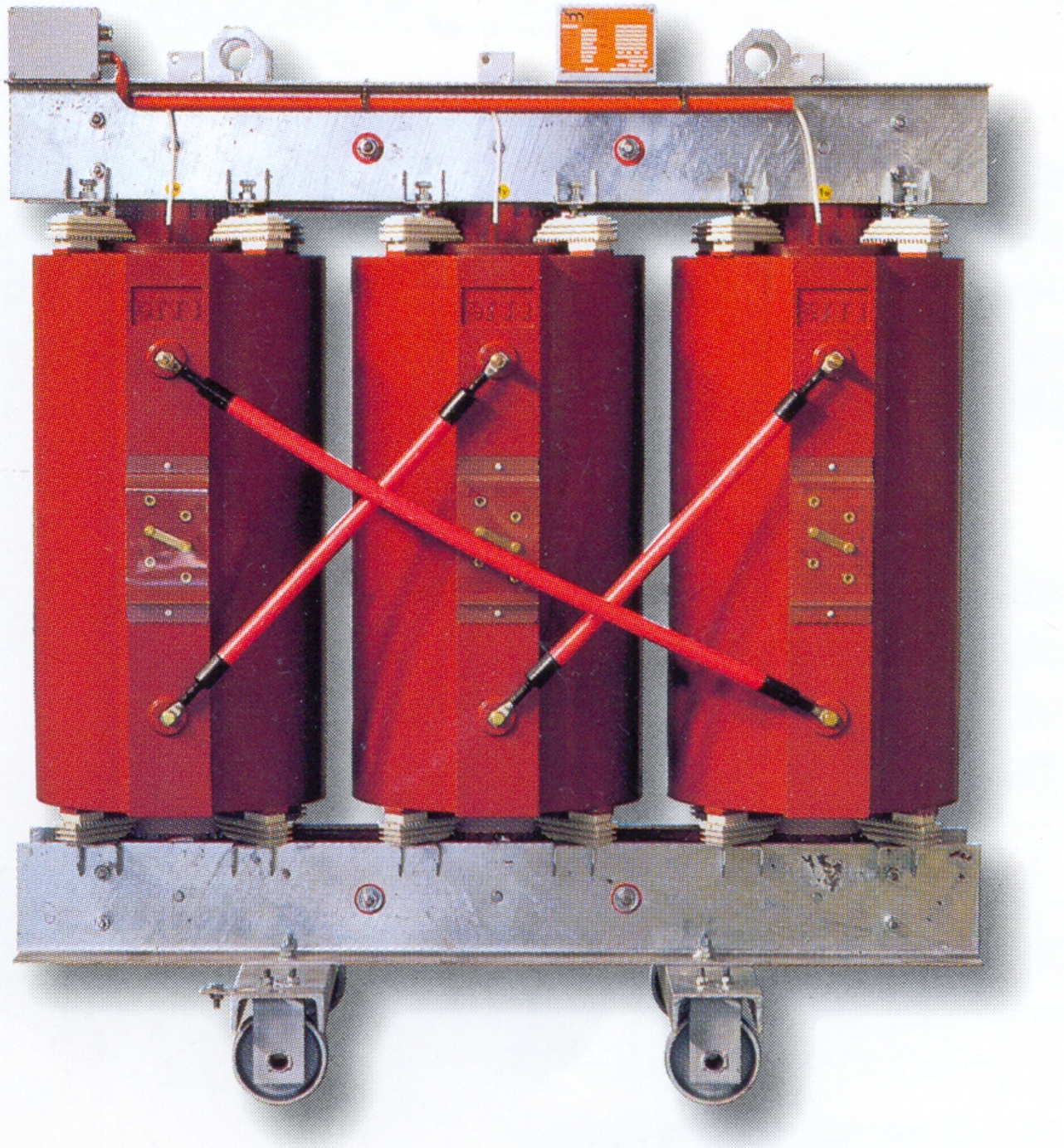
6 / 0,4 kV





1500 kVA

22 / 1,5 kV



400 kVA

22 / 0,4 kV

1000 kVA

22 / 0,4 kV

250 – 630 kVA

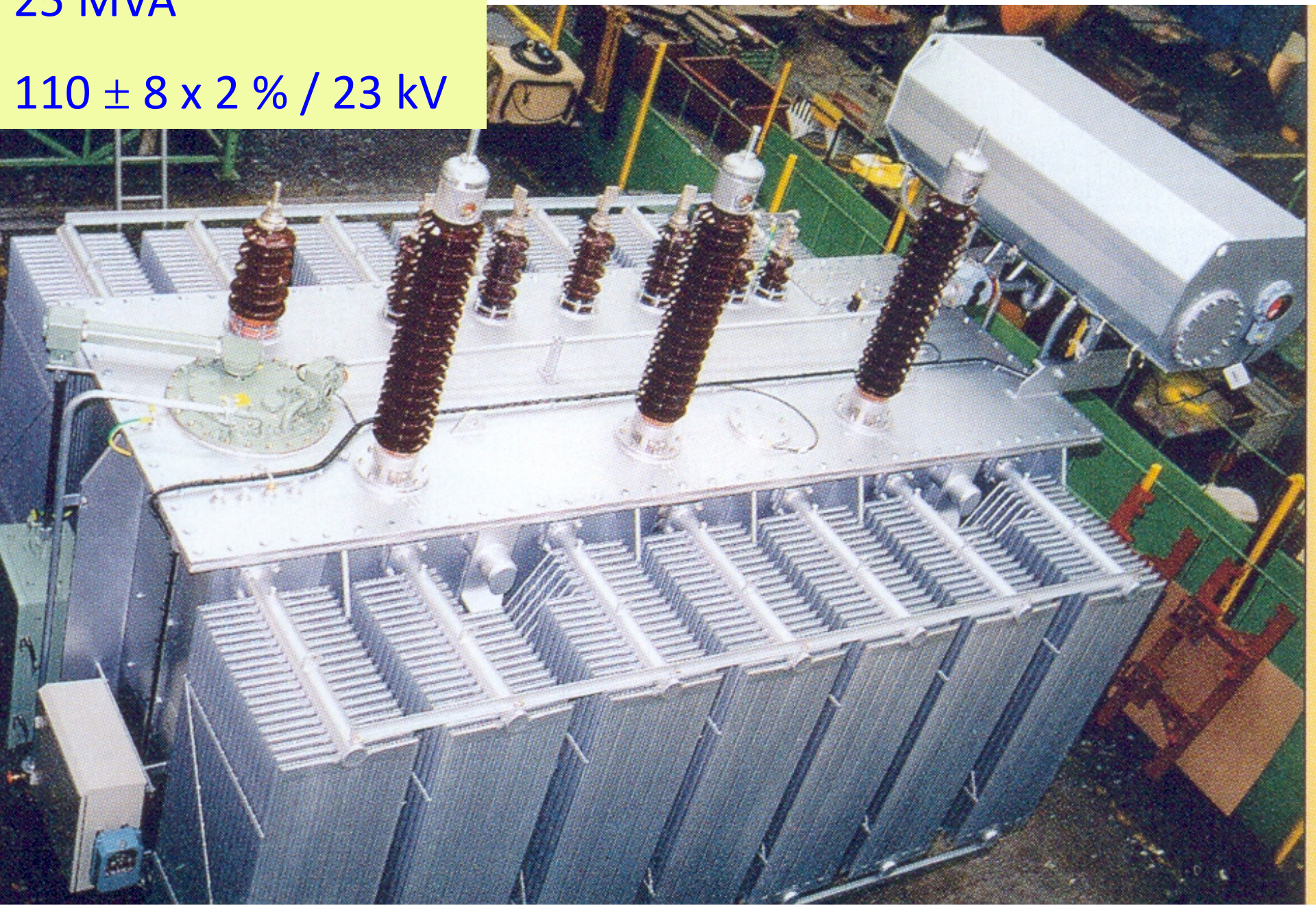
22 kV

50 – 1000 kVA

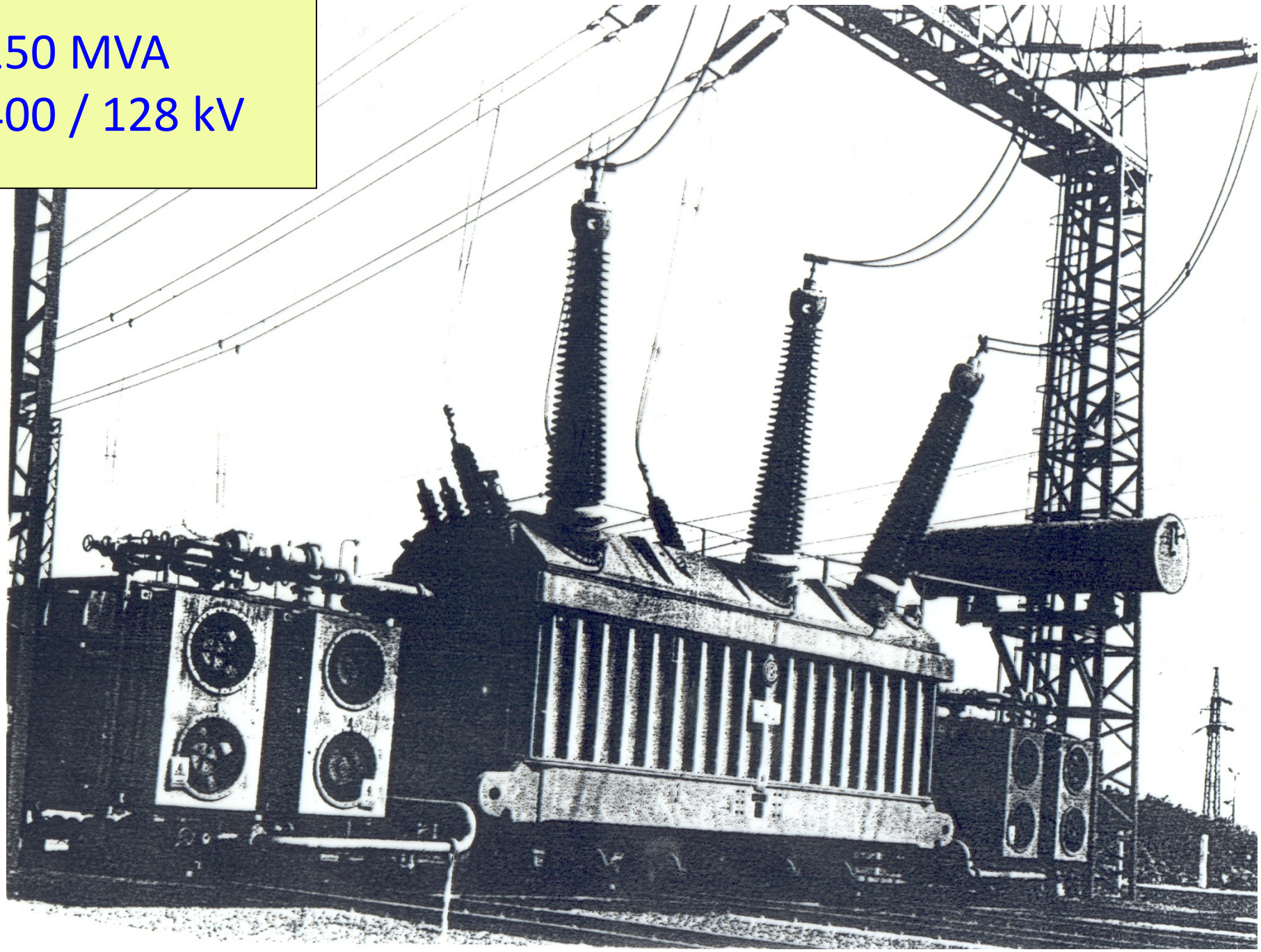
35 kV

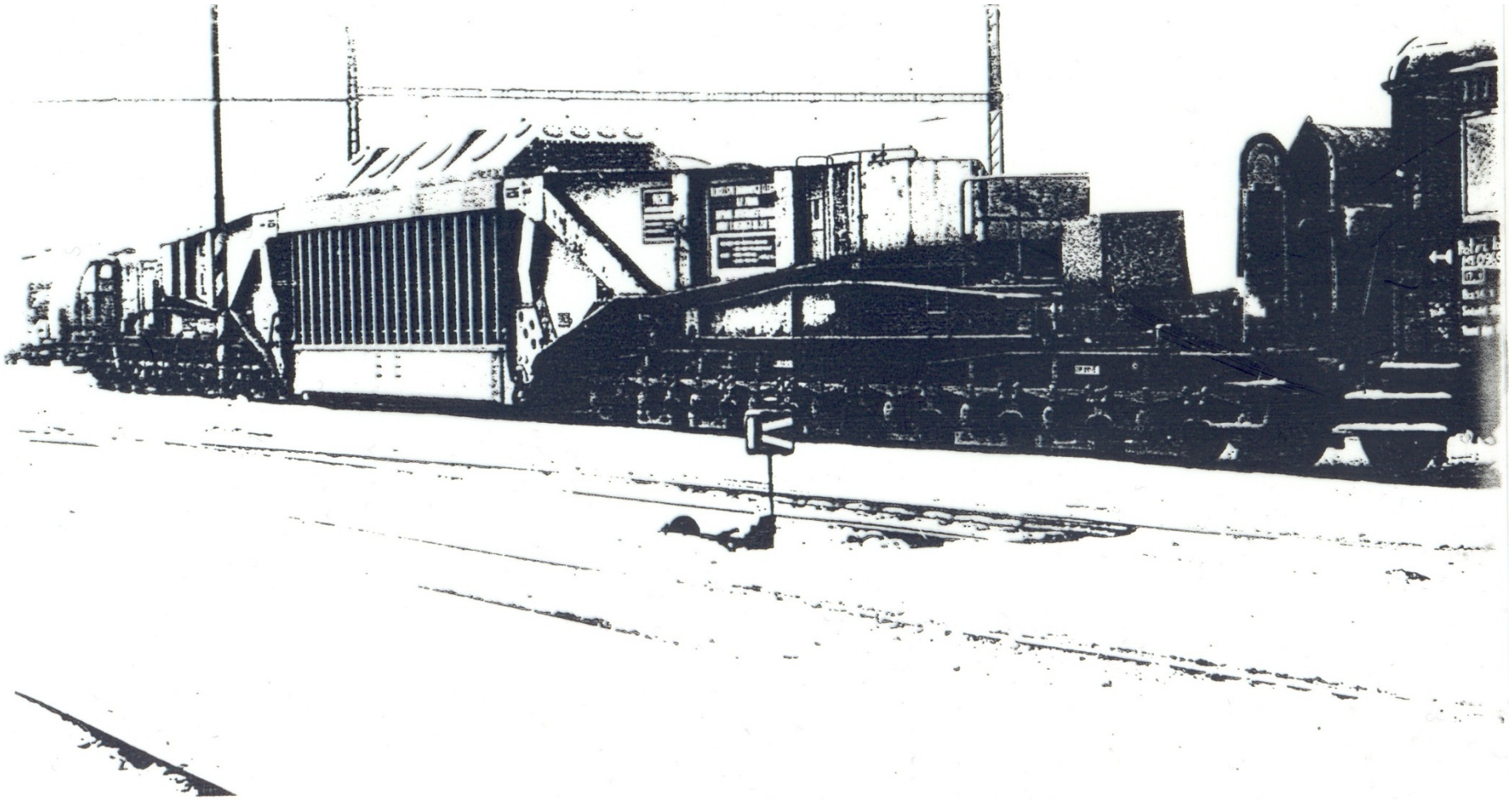
25 MVA

110 ± 8 x 2 % / 23 kV



250 MVA
400 / 128 kV





T24